## THE FOURTEENTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE fourteenth annual meeting of the Society was held in New York City on Friday and Saturday, December 27–28, 1907. Two sessions were held on Friday and one on Saturday morning. On Friday evening about thirty members gathered at the informal dinner and smoker which has long been a feature of the Society's meetings. The attendance at the three sessions included the following fifty-three members:

Professor Joseph Allen, Professor William Beebe, Mr. William Betz, Professor G. A. Bliss, Professor C. L. Bouton, Professor Joseph Bowden, Professor E. W. Brown, Dr. Emily Coddington, Professor F. N. Cole, Professor T. C. Esty, Professor A. S. Gale, Dr. F. L. Griffin, Professor C. O. Gunther, Professor J. G. Hardy, Professor H. E. Hawkes, Dr. L. I. Hewes, Mr. H. R. Higley, Dr. A. A. Himowich, Dr. J. G. Hun, Professor W. H. Jackson, Mr. S. A. Joffe, Professor C. J. Keyser, Professor P. A. Lambert, Mr. W. D. Lambert, Professor W. W. Landis, Dr. N. J. Lennes, Dr. G. H. Ling, Dr. W. R. Longley, Mr. E. B. Lytle, Professor James Maclay, Professor Max Mason, Mr. C. N. Moore, Dr. R. L. Moore, Professor Frank Morley, Professor Richard Morris, Professor G. D. Olds, Professor Anna H. Palmié, Dr. H. B. Phillips, Professor James Pierpont, Miss Amy Rayson, Mr. H. W. Reddick, Professor R. G. D. Richardson, Professor P. F. Smith, Professor J. H. Tanner, Mr. C. A. Toussaint, Mr. M. O. Tripp, Professor H. W. Tyler, Professor Oswald Veblen, Mr. H. E. Webb, Professor H. S. White, Professor E. B. Wilson, Professor J. E. Wright, Professor J. W. Young.

President H. S. White presided at the morning sessions. Vice-President P. F. Smith took the chair on Friday afternoon. The Council announced the election of the following persons to membership in the Society: Mr. Charles Ammerman, McKinley High School, St. Louis, Mo.; Dr. C. S. Atchison, Williams College; Mr. B. H. Camp, Wesleyan University; Professor W. M. Carruth, Hamilton College; Mr. G. R. Clements, Williams College; Professor Julia T. Colpitts, Iowa State College; Professor J. N. Ivey, Tulane University; Professor W. H. Jackson, Haverford College; Mr. W. C. Krath-

wohl, Columbia University; Professor Murray Macneill, Dalhousie University; Mr. C. N. Moore, Harvard University; Professor Maria M. Roberts, Iowa State College; Mr. E. W. Sheldon, Yale University. Seven applications for membership in the Society were received.

In response to the invitation of Professor E. J. Townsend, it was decided to hold the next summer meeting of the Society at the University of Illinois. The date of the next annual meeting was fixed as Tuesday and Wednesday, December 29–30.

The reports of the Treasurer and Auditing Committee will appear in the Annual Register. The membership of the Society has increased during the past year from 554 to 580, including at present 52 life members. The number of papers presented at all meetings during the year was 166. The total attendance of members was 398. The Treasurer's report shows a balance of $6,081.82, of which $2,969.40 is credited to the life membership fund. Sales of the Society's publications during the year amounted to $1,534.

Several important changes have occurred in the past year in the administrative and editorial offices. Dr. Dennett retired in February from the Treasurer's office, and was succeeded by Professor Tanner. Professor Bôcher has succeeded Professor Moore as editor in chief of the *Transactions*. At the close of the year Professor Alexander Ziwet retired from the Committee of Publication, of which he had been a most valued member since 1893. The vacancy in the Committee was filled by the election of Professor Virgil Snyder.

At the annual election, which closed on Saturday morning, the following officers and members of the Council were chosen:

*Vice-Presidents*, Professor JAMES HARKNESS,
Professor G. A. MILLER.
*Secretary*, Professor F. N. COLE.
*Treasurer*, Professor J. H. TANNER.
*Librarian*, Professor D. E. SMITH.

*Committee of Publication*,
Professor F. N. COLE,
Professor D. E. SMITH,
Professor VIRGIL SNYDER.

*Members of the Council to serve until December*, 1910,

Professor R. E. ALLARDICE, Professor G. D. OLDS,
Professor MAX MASON, Professor M. B. PORTER.

The following papers were read at this meeting:

(1) Dr. F. L. GRIFFIN: "Certain families of central orbits with a constant apsidal angle."

(2) Dr. F. L. GRIFFIN: "On the non-existence of certain types of periodic solutions in the problem of three bodies."

(3) Professor E. O. LOVETT: "On a problem in mechanics. Note II."

(4) Professor E. O. LOVETT: "On a class of periodic solutions in the problem of four bodies."

(5) Professors OSWALD VEBLEN and J. W. YOUNG: "A set of assumptions for projective geometry."

(6) Professor FRANK MORLEY: "The transformation of a Clifford configuration into itself."

(7) Professor R. G. D. RICHARDSON: "Lebesgue improper integrals."

(8) Professor E. W. BROWN: "The motion of the moon relative to a moving plane of reference."

(9) Professor E. W. BROWN: "The development of the infinite determinant."

(10) Dr. ARTHUR RANUM: "Groups of singular matrices."

(11) Professor J. G. HARDY: "Curves in a space of $n$ dimensions."

(12) Dr. A. B. COBLE: "A reduction of the problem of solving a quintic to Klein's 'problem of the $A$'s' by means of invariant theory."

(13) Dr. L. I. HEWES: "The necessary and sufficient condition that an ordinary differential equation shall admit a conformal group."

(14) Professor J. I. HUTCHINSON: "Hermitian forms with zero determinant."

(15) Professor E. B. WILSON: "On the uniform rotation of a homogeneous chain about a vertical axis."

(16) Professor E. B. WILSON: "On the theory of double products and strains in hyperspace.

(17) Professor MAX MASON: "Note on implicit functions."

(18) Professor H. E. HAWKES: "On the equivalence of families of bilinear forms."

(19) Professor G. A. BLISS: "An existence theorem for a partial differential equation of the first order which is non-analytic."

In the absence of the authors Professor Hutchinson's paper was presented by Professor J. W. Young, and the papers of

Professor Lovett, Dr. Ranum, and Dr. Coble were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Bertrand has discussed laws of central force for which every orbit is a closed curve.* In reality, however, he proved a far more general theorem than he stated, viz., that there are only two laws for which all the trajectories have a constant angle $\Theta$. In Dr. Griffin's first paper, after noting this fact, consideration is given to those laws for which $\Theta$ is constant in certain families of orbits. It is shown that, while (for $n \neq 1, 2$) there is no law for which $\Theta = \pi/n$ in all the orbits, yet (for $n$ arbitrary, rational or irrational) there exist laws of force for which $\Theta = \pi/n$ in a double infinitude of orbits, viz., in those families, the average of the $n$th powers of the apsidal distances of whose orbits is an arbitrary constant.

2. In a recent paper† Professor Moulton has proved the existence of a certain class of periodic plane orbits of three bodies. In this note Dr. Griffin shows that of two types closely related to these there are no orbits in three dimensions. The physical reasons for this non-existence are pointed out.

3. Professor Lovett considers the most general case of a problem studied in a paper presented to the Society at its last summer meeting (See BULLETIN, volume 14 (1907), page 57). Two differential systems $F$ and $G$ are to be integrated. The system $F$ in its most general form is integrated, and the complete integration of $G$ is achieved for those cases in which the conditions of integrability of $F$ are realized.

As to the parameters, they must satisfy but two of the relations found in the former paper, namely $\alpha : \gamma : \delta = 9 : -45 : 40$. As regards the forces, we have the curious result that there appear no other forms than those originally found by Darboux and Halphen in the case of conic trajectories.

4. In a recent number of the *Annali di Matematica* Mr. G. Pavanini‡ has constructed a new category of periodic solutions

* *Comptes rendus*, vol. 77, pp. 849–53.

† *Transactions*, vol. 7, pp. 537–577.

‡ Pavanini, "Sopra una nuova categoria di soluzioni periodiche nel problema dei tre corpi," *Annali di Matematica*, 3d series, vol. 13 (1906), pp. 179–203.

of the problem of three bodies. It is the object of Professor Lovett's second note to write out the corresponding solutions for those cases of the problem of four bodies which consist of three finite bodies in motion according to one of the lagrangian exact solutions of the problem of three bodies and a fourth body infinitesimally small.

5. The paper of Professors Veblen and Young contains a completely independent* set of assumptions for projective geometry, in which points and undefined classes of points called lines have been taken as the undefined elements. The assumptions are so arranged that a certain group of eight characterize what may be called general projective spaces, *i. e.*, spaces in which the points can be represented by homogeneous coordinates which are elements of a finite or infinite number system, in which the operation of multiplication may or may not be commutative. On adding to this group a ninth assumption (from which the so-called fundamental theorem of projective geometry can be derived), we obtain a set of assumptions which characterize what we call properly projective spaces, namely spaces in which the points can be represented by coordinates which are elements of a commutative number system, *i. e.*, of a finite or infinite field.

Spaces in which the coordinates are elements of modular and non-modular fields are then distinguished by a further assumption. Finally it is shown how by adding other assumptions one may arrive at categorical and completely independent sets of assumptions for the projective geometries in which the coordinates are on the one hand the ordinary real numbers, and on the other hand, the ordinary complex numbers.

6. Professor Morley's paper is in abstract as follows: Clifford's theorem on $p$ lines of a plane (Works, page 38) gives a regular configuration $T_p$ of $2^{p-1}$ points and as many circles; on each point are $p$ circles and on each circle are $p$ points. Bifocal curves of class $n$, on $3n-3$ given lines, set up a Cremona transformation between the two real foci. This gives in certain cases a transformation of the points of a Clifford configuration into themselves. Thus for $T_6$ the transformation is

$$xy = m + \mu t, \quad \bar{x} + \bar{y} + \bar{a} = t(x + y + a).$$

The paper will be offered to the *Transactions*.

* Ordinally independent sets have been given before; but so far as the authors are aware this is the first completely independent set.

7. In a previous paper, Professor Richardson has shown that the existence of the double and iterated integrals of any function is a sufficient condition for their equality. The present discussion covers the case where one only of the integrals is assumed to exist. By means of the notion of measure of an aggregate introduced by Borel, Lebesgue has defined * an integral which includes the ordinary (Riemann) integral as a special case. As far as is known, this integral exists for every limited function. By means of a generalization of the Lebesgue integral to the case of an unlimited function, it is shown that the existence of the double integral of any function $f(x, y)$ in a two-dimensional domain $T$ is a sufficient condition for the equality

$$\int_T f(x, y)dT = L_Y dy \underline{\int}_X^{\overline{}} f(x, y)dx,$$

where the symbol $L$ denotes the Lebesgue integral. The existence of the integrals $\int_Y \underline{\int}_X^{\overline{}} f$, $\int_Y \underline{\int}_X^{\overline{}} |f|$ is a sufficient condition for the existence of the Lebesgue double integral and for the equality

$$\int_Y \underline{\int}_X^{\overline{}} = L_T f.$$

Various other relations of a more general nature are derived and the results extended to the case where the domain $T$ is unlimited.

8. Methods for partially computing the motion of the moon relative to a moving plane of reference have been given by Adams, Hill, Radau, and others. It is shown in Professor Brown's first paper that the general formulas for referring the motion to moving axes give a simple disturbing function for the general case where the periodic as well as the secular terms in the motion of the ecliptic are included.

9. In Professor Brown's second note the methods given by Hill (*Acta Mathematica*, volume 8) are replaced by the well-known formula for the development of a determinant in powers

* Leçons sur l'intégration et la recherche des fonctions primitives, Paris, Gauthier-Villars, 1904.

of the constituents of the leading diagonal. The summing of the resulting series is made by a method which avoids the use of conditionally convergent series.

10. In order that a set of singular $n$-ary matrices (linear substitutions with vanishing determinants) may form a group in the ordinary sense (not in the broader sense in which the word is used by Frobenius and Schur in the *Berliner Sitzungsberichte* for 1906, I, page 209), all its matrices must have the same rank $r$, and their characteristic equations must have the same number of zero roots, viz., $n - r$. Every such group is reducible; in its reduced form every matrix consists of an ordinary $r$-ary matrix bordered with zeros. A singular matrix $S$ will belong to some group if, and only if, the invariant factors of its characteristic equation corresponding to zero roots are linear. $S$ belongs to a group of finite order if, and only if, the non-zero roots of its characteristic equation are roots of unity and the invariant factors of its characteristic determinant are all linear. $S$ is the identity matrix of some group (idempotent) if, and only if, the roots of its characteristic equation are all equal to 0 or 1 and the invariant factors of its characteristic determinant are linear. Every matrix of rank $n$ or $n - 1$ belongs to some group.

11. In a paper published in the *American Journal of Mathematics*, volume 24, Professor Hardy derived for curves in a space of four dimensions formulas corresponding to the Serret-Frenet formulas for curves in a three-dimensional space. The present paper has for its object the derivation of Serret-Frenet formulas for a curve of $n - 1$ curvatures in a space of $n$ dimensions. The method employed is the kinematic method used by Darboux in his Théorie des surfaces, volume 1, chapter 1.

12. Except for the preliminary adjunction of the square root of the discriminant, Klein's "problem of the $A$'s," which depends on two essential parameters, requires for its solution all the irrationalities and transcendents necessary in the solution of the general quintic. Dr. Coble shows that all the rational processes required to reduce the solution of the quintic to the problem of the $A$'s can be set forth by invariantive methods. A form problem in which the known quantities are the values of the invariants is developed. The underlying $G_{120}$ is a Cre-

mona group. The one necessary transformation of the quintic is accomplished by means of the linear covariants.

13. Given the differential equation $dy/dx = a(x, y)$, the problem of Dr. Hewes's paper is to determine the necessary and sufficient condition that this equation shall admit a one-parameter continuous conformal group. It is found that under the infinite group of all conformal transformations there exist, in connection with the differential equation above, invariant functions of $A$ and its partial derivatives. There are two of the third order, from which two more of the fourth order are readily deduced. The necessary and sufficient condition is then that either of these invariants of the third order and the corresponding derived one of the fourth order are functions of the same harmonic function. If this condition is satisfied the group is uniquely determined.

14. Professor Hutchinson's paper appeared in full in the February BULLETIN.

15. The problems in the equilibrium of strings or chains which have been solved since the days when Bernoulli first investigated the catenary seem to be almost wholly those in which the field of force is rectilinear. One reason for this is probably the fact that fields other than the parallel and central fields lead to differential equations which are not linear. Professor Wilson treats the problem of equilibrium in a field compounded of gravity and a centrifugal force perpendicular to a vertical axis. This problem is readily recognized in the case of a string suspended at one end and forced to rotate at a uniform rate about a vertical axis through the point of suspension. Although the differential equation is not linear, it is of such form that it may readily be discussed qualitatively. At points considerably removed from the lowest point of the chain an approximate differential equation of the Bessel type may be obtained, and the qualitative discussion shows that in case the lower end of the string is free the configuration in the neighborhood of that point is also represented (qualitatively) to a remarkable degree by the approximate equation. One curious consequence brought out by the investigation is the existence of a rapidly increasing set of numbers $\alpha, \beta, \gamma \cdots$ such that, if the length $l$ of the string is less than $\alpha$, the string cannot depart from the vertical; whereas

if $\alpha < l < \beta$, there is one position of equilibrium and only one, if $\beta < l < \gamma$, there are two and only two positions of equilibrium, and so on. The paper will appear in the *Annals of Mathematics*, volume 9 (1908).

16. In the paper on the theory of double products and strains in hyperspace Professor Wilson presents: first, an account of so much of the late Professor Gibbs's lectures on multiple algebra as is essential to the understanding of his theory of dyadics; and second, some applications to the theory of strains and collineations in relation to involutory strains and collineations carried out by means of Gibbs's algebra. The results of this second part of the paper may be summarized briefly as follows: The determination of the square roots of the idemfactor by means of the properties of the equation of least degree; the determination of the necessary and sufficient conditions that a dyadic be resolvable into the product of two square roots of the idemfactor, and the correlation of these results with the theory of reflections in connection with unimodular strains and collineations; the introduction of certain invariant dyadics associated with any given dyadic and their application; the result that, if the Hamilton-Cayley equation of a dyadic is of least degree, the dyadic is resolvable into the product of three square roots of the idemfactor, of which the first is of the simplest type; the determination of the necessary and sufficient conditions that any dyadic be resolvable into the product of three square roots of the idemfactor.

17. The paper of Professor Mason is concerned with the determination of real implicit functions by an equation $f(x, y)=0$ when the partial derivatives of $f$ of order less than $n$ vanish at the point in question. Existence theorems are obtained with the aid of Dini's theorem, after applying certain transformations similar to those used in resolving higher singularities of an algebraic curve.

18. From the properties of matrices considered as linear vector operators, Professor Hawkes proved Weierstrass's theorems, that the necessary and sufficient condition for the equivalence of two non-singular families of bilinear forms is that their elementary divisors are equivalent.

19. For a partial differential equation of the first order

$$F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0,$$

in which the first member is an analytic function of its five arguments, there exists in general through an analytic curve an analytic surface $z = z(x, y)$ which satisfies the equation. In the paper of Professor Bliss it is shown that a similar theorem is true when the function $F$ is required to have continuous first and second derivatives only. The proof is made with the help of the theory of characteristic curves and the existence theorems for a set of ordinary differential equations

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \cdots, x_n), \quad \cdots, \quad \frac{dx_n}{dt} = f_n(x_1, x_2, \cdots, x_n).$$

The relation between the characteristic curves and integral surfaces is well known when everything is analytic. But in order to work the other way and derive existence theorems from the properties of characteristic curves, it is necessary to use the theorems on the differentiability of solutions of a system of ordinary equations with respect to the constants of integragration. These theorems seem to have been overlooked in this connection.

F. N. COLE,
*Secretary.*

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## THE DECEMBER MEETING OF THE CHICAGO SECTION.

THE twenty-second regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago on Monday, Tuesday and Wednesday, December 30–31, 1907, and January 1st, 1908, in connection with the fifty-eighth convocation of the American association for the advancement of science. The great gathering of scientists in other lines doubtless attracted unusual numbers of mathematicians, resulting in a wide representation of members and the largest attendance ever recorded at any meeting of the Society.

Monday afternoon and Tuesday morning were devoted to meetings with Sections A and D of the American association, for discussion of the teaching of mathematics to engineering

students. These meetings proved of so great interest that an additional half-day was set apart for the informal discussion, and this session was finally brought to a most delightful climax by the vice-presidential address of Professor Edward Kasner of Section A, on "Geometry and mechanics," which proved equally fascinating to both engineers and mathematicians. A detailed report of these joint meetings, with abstracts of the papers, appears elsewhere in the BULLETIN, and the papers will be published in full in *Science* during the year.

The sessions on Monday morning, Wednesday morning, and Wednesday afternoon were given to the reading of papers. Professor Heinrich Maschke, Vice-President of the Society, presided at the first session, and Professor E. B. Van Vleck, Chairman of the Section, relieved at times by Professor L. E. Dickson and Professor H. B. Newson, presided at the other sessions.

The officers of the Section elected for the ensuing year were: Professor G. A. Miller, Chairman; Professor H. E. Slaught, Secretary, and Professor D. R. Curtiss, third member of the programme committee.

The promotion of acquaintance and good fellowship was an unusually prominent feature of the meetings, owing to the complete arrangements for entertainment and especially to the subscription dinner at the Hotel Del Prado, where just one hundred mathematicians and engineers were gathered. It has long been the custom on these occasions to send postal card greetings to absent members — a custom inaugurated by Professor E. H. Moore — and in this instance the hundred greetings were sent to him, on board ship departing for his year's vacation.

The attendance at the several sessions included the following eighty-four members of the Society:

Professor D. P. Bartlett, Professor S. M. Barton, Professor W. W. Beman, Dr. G. D. Birkhoff, Professor O. Bolza, Professor W. C. Brenke, Professor W. E. Brooke, Mr. Thomas Buck, Professor W. H. Bussey, Professor D. F. Campbell, Professor H. E. Cobb, Mr. E. H. Comstock, Professor C. E. Comstock, Professor D. R. Curtiss, Dr. H. N. Davis, Professor E. W. Davis, Professor S. C. Davisson, Professor L. E. Dickson, Professor J. F. Downey, Professor L. W. Dowling, Mr. Arnold Dresden, Professor H. T. Eddy, Professor C. C. Engberg, Mr. E. B. Escott, Professor H. B. Evans, Professor T. M. Focke, Mr. R. M. Ginnings, Miss Harriet E. Glazier,

Professor B. F. Groat, Professor A. G. Hall, Professor G. B. Halsted, Professor Harris Hancock, Dr. Charles Haseman, Professor C. N. Haskins, Professor A. S. Hathaway, Professor E. R. Hedrick, Mr. T. H. Hildebrandt, Mr. F. H. Hodge, President C. S. Howe, Professor E. V. Huntington, Professor Edward Kasner, Professor O. D. Kellogg, Professor Kurt Laves, Dr. A. C. Lunn, Mr. W. A. Luby, Professor C. R. Mann, Mr. W. D. MacMillan, Professor J. L. Markley, Professor Heinrich Maschke, Professor G. A. Miller, Professor F. R. Moulton, Professor G. W. Myers, Dr. L. I. Neikirk, Professor B. L. Newkirk, Professor H. B. Newson, Mr. E. W. Ponzer, Professor Charles Puryear, Professor N. C. Riggs, Professor D. A. Rothrock, Mr. A. R. Schweitzer, Miss Ida M. Schottenfels, Professor G. T. Sellew, Professor J. B. Shaw, Mr. R. L. Short, Dr. C. H. Sisam, Mr. C. G. Simpson, Professor E. B. Skinner, Professor H. E. Slaught, Professor C. S. Slichter, Professor S. E. Slocum, Mr. E. R. Smith, Professor A. W. Smith, Mr. E. H. Taylor, Professor E. J. Townsend, Dr. A. L. Underhill, Professor E. B. Van Vleck, Professor C. A. Waldo, Professor C. B. Williams, Professor D. T. Wilson, Professor E. J. Wilczynski, Professor F. S. Woods, President R. S. Woodward, Mr. A. E. Young, Professor J. W. A. Young, Professor Alexander Ziwet.

The following papers were read:

(1) Dr. J. C. MOREHEAD: "The rapid computation of power residues."

(2) Dr. C. H. SISAM: "On the inflectional tangents to a quartic curve."

(3) Dr. C. H. SISAM: "Some loci connected with plane curves, II."

(4) Professor J. B. SHAW: "Standard forms of certain types of Peirce algebras."

(5) Professor G. A. MILLER: "On the multiple holomorphs of a group."

(6) Professor L. W. DOWLING: "On the generation of plane quintics with ordinary double points."

(7) Professor E. J. WILCZYNSKI: "Projective differential geometry of curved surfaces, III."

(8) Dr. CHARLES HASEMAN: "Some boundary problems in the theory of functions."

(9) Dr. A. R. CRATHORNE: "Hilbert's invariant integral in the general isoperimetric problems."

(10) Mr. E. B. ESCOTT: "The converse of Fermat's theorem."

(11) Professor W. C. BRENKE: "Convergence, summability and differentiability of trigonometric series."

(12) Mr. A. W. SCHWEITZER: "On Moore's 'Tactical memoranda.'"

(13) Dr. G. D. BIRKHOFF: "On a certain existence and oscillation theorem."

(14) Professor E. H. MOORE: "Note on a form of general analysis."

(15) Professor L. E. DICKSON: "Reduction of families of quadratic forms in a general field."

(16) Professor L. E. DICKSON: "On commutative linear groups."

(17) Mr. H. E. BUCHANAN: "Note on the convergence of a sequence of functions of a certain type."

(18) Dr. A. L. UNDERHILL: "Note on the calculus of variations."

(19) Professor E. R. HEDRICK: "Note on the structure of point sets" (preliminary communication).

(20) Professor C. N. HASKINS: "Note on the generalized law of the mean."

(21) Professor F. R. MOULTON: "On certain relations, due to tidal friction, in the motion of two mutually attracting bodies."

(22) Professor A. S. HATHAWAY: "Motion of three bodies with fixed center of gravity."

Mr. Buchanan was introduced by Professor Moulton. In the absence of the authors the papers of Dr. Morehead, Dr. Crathorne, and Professor Moore were read by title. Abstracts of the papers follow below; the numbering is the same as that attached to the titles in the above list.

1. Dr. Morehead's paper describes a means of solving easily the congruence $a^{kn}x \equiv b \pmod{m}$ for successive values of $k$. The solutions $x$ are $n$th power residues or non-residues, modulo $m$, according as $b$ is an $n$th power residue or non-residue. The power residues, modulo $m$, of all degrees may be obtained by the solution of $\phi(m)$ (the totient function) congruences of the above form. The case $a = 2$ is the most important, because of its simplicity.

2. In this paper Dr. Sisam proves that the inflectional tangents to an arbitrary quartic curve form the complete set of tangents common to a curve of class four and one of class six. The curve of class four in question is the envelope of the lines meeting the quartic in equianharmonic points; the one of class six is the envelope of the lines meeting the quartic in harmonic points.

3. In a paper previously presented * Dr. Sisam discussed the locus of the points of intersection of the tangents to a curve when the lines joining their points of tangency touch a second given curve. In the present paper, Plücker's numbers for these loci are completely determined, several important special cases are considered, and the dual loci are discussed.

4. The following types of Peirce algebras (that is, non-quaternion algebras with no skew units) have previously been considered and the subtypes exhibited:

$(i, i^2, \cdots, i^{r-1})$, deficiency zero, by Benjamin Peirce.
$(i, j^2, \cdots, j^{r-2})$, deficiency unity, by Benjamin Peirce.
$(i, i^2, j, \cdots, j^{r-3})$, deficiency two, by Starkweather.
$(i, j, ij, j^2, \cdots, j^{r-3})$, deficiency two, by Starkweather.
$(i, j, k, k^2, \cdots, k^{r-3})$, deficiency two, by Starkweather.

It is the purpose of Professor Shaw's paper to consider further types and reduce the same to their simplest forms; in particular the types $(i, i^2, \cdots, i^m, j, \cdots, j^{r-m-1})$, $(i, j, \cdots, j^{m-1}, k, \cdots, k^{r-m-1})$, $(i, i^2, \cdots, i^m, j, ij, \cdots, j^{r-m-2})$, $(i, i^2, \cdots, i^m, j, ij, j^2, ij^2, j^3, \cdots, j^{r-m-3})$.

5. In considering the holomorph of an abelian group $G$ of odd order, Burnside proved a theorem under the assumption that $G$ is a characteristic subgroup of its holomorph (Theory of groups of finite order, page 238). Professor Miller proves that this assumption is superfluous, by establishing the theorem that an abelian group of odd order is always a characteristic subgroup of its holomorph. He considers the general question, under what condition an abelian group is a characteristic subgroup of its holomorph. Some of his results are expressed in the following theorems: If the order of an abelian group is not divisible by 8, it is a characteristic subgroup of its holomorph. If an abelian group $K$ of order $2^m$ contains only one largest in-

* BULLETIN, vol. 13, p. 277.

variant exceeding $2^3$ and if its holomorph contains another invariant subgroup $K_1$ which is similar to $K$, then $K$, $K_1$ will generate a group whose commutator subgroup is of order 2. When $K$ contains only one second largest invariant, its holomorph contains exactly three other invariant subgroups which are similar to $K$; but its holomorph contains only one other invariant subgroup similar to $K$ when $K$ contains more than one second largest invariant. The holomorph of any abelian group $K$ cannot involve more than four invariant subgroups which are similar to $K$.

6. Castelnuovo (*Mathematische Annalen*, volume 44), has proved that every plane involution is rational, *i. e.*, that every such involution may be set up by a net of curves. Given, therefore, any involution $I_n$ of order $n$, it is possible to find a net of curves such that any two curves of the net intersect in a set of points of the involution, and any two sets of points determine a curve of the net. The plane may be considered as containing two sorts of elements, the point sets of $I_n$ and the curves of the net. The geometry in such a plane includes the discussion of all plane curves which can be the loci of singly infinite series of point sets of $I_n$ or the envelopes of singly infinite series of curves of the net.

Professor Dowling considers the $I_{6-d}$ set up by a net of cubics having three collinear points in common and $d$ other points, where $d \leqq 4$. Quintic curves possessing simple double points appear among the involution curves. The converse problem, whether a given quintic, can be considered as an involution curve in an $I_{6-d}$, is answered in the affirmative.

Other involutions are considered with a view to applying the notion of an involution in the plane to the generation of any algebraic plane curve.

7. In a previous paper Professor Wilczynski has shown that the equation of every non-ruled surface can be developed, in the vicinity of an ordinary point, in the canonical form

$$(1) \qquad z = xy + \tfrac{1}{6}(x^3 + y^3) + \tfrac{1}{24}(Ix^4 + Jy^4) + \cdots,$$

where $I$, $J$ and all further coefficients are absolute invariants of the surface. No development of this form exists for a ruled surface. It is the purpose of the present paper to find a

canonical development for this exceptional case and to discuss it in detail. The canonical form found is

$$(2)\quad z = xy + x^3 + x^5 + Ax^4y + Bx^3y^2 + \text{terms of the sixth order,}$$

where $A$ and $B$ are absolute invariants. Certain exceptional cases are also discussed in which this new development also breaks down.

The question as to the meaning of the canonical development (2), *i. e.*, primarily as to the geometric significance of the tetrahedron of reference which gives rise to this development, is then taken up and completely answered. A number of new and important results are found on the way. It is found that there exists a single infinity of cubic scrolls which have contact of the fourth order with the given ruled surface at the given point. Each of these scrolls has a nodal straight line with two pinch points upon it. The locus of the nodal lines is the osculating hyperboloid. The locus of the pinch points is a twisted cubic curve upon the osculating hyperboloid.

Among this single infinity of cubic scrolls there is one whose pinch points coincide (a Cayley cubic scroll). Thus there exists one and only one Cayley cubic scroll which has contact of the fourth order with a given ruled surface at a given point. The pinch point, nodal line, and singular tangent plane of this surface are respectively a vertex, an edge, and a face of the canonical tetrahedron. Thereby the tetrahedron is completely determined. For the two edges which meet at the given point of the surface are the two asymptotic tangents of the surface at that point.

There is thus a unique Cayley cubic scroll associated with every point of a ruled surface. The question arises as to the relation between those which belong to points of the same generator. It is found that this relation is as follows: the locus of the pinch points of the osculating Cayley cubic scrolls, which belong to the various points of any generator, is a twisted cubic curve which is situated on the osculating hyperboloid, and which intersects the generator in its flecnodes.

8. In this paper Dr. Haseman first shows how to prove the existence of a regular analytic function of a complex variable $f(z) = u(x, y) + iv(x, y)$ on a multiply connected as well as on a simply connected surface, where the relation

$$a(s)\, u(s) + b(s)\, v(s) + c(s) = 0$$

between the boundary values $u(s)$ and $v(s)$ of the real and imaginary parts respectively of $f(z)$ is given, and where $a(s)$, $b(s)$, $c(s)$ may each have a finite number of finite discontinuities. This is done by choosing a regular analytic function $g(z)$ in the given region such that $f^*(z) = f(z)g(z)$, where $f^*(z)$ may be gotten from the relation $a^*(s)\,u^*(s) + b^*(s)\,v^*(s) + c^*(s) = 0$, and where the coefficients in the relation are all continuous throughout.

A number of special cases arise from this theory. For example, a regular analytic function $f(z)$ may be found, where the real part $u(s)$ is given for certain segments of the bounding curve of the given region and the imaginary part $v(s)$ for the rest of the boundary.

A similar problem may be solved for multiply connected surfaces; for example, we may find $f(z)$ for the region between two concentric circles, where $u(s)$ is given on the outer circle and $v(s)$ on the inner circle.

It is next shown how to find a regular analytic function of a complex argument $f_a(z)$ exterior to a given region and another function of the same character $f_j(z)$ interior to the region, when the relation $f_a[\sigma(s)] = c(s)f_j(s)$ is given between the boundary values $f_a(\sigma)$ and $f_j(s)$ of our functions at the points $\sigma = \sigma(s)$ and $s$ respectively, where $\sigma(s)$ is a regular differentiable function of $s$ and where $c(s)$ may have a finite number of finite discontinuities.

The required functions are found by choosing two other functions $g_a(z)$, $g_j(z)$ of the same characters such that $f_a^*(z) = f_a(z)g_a(z)$ and $f_j^*(z) = f_j(z)g_j(z)$, in which the starred functions are gotten from the relation $f_a^*[\sigma(s)] = c^*(s)f_j^*(s)$, where $c^*(s)$ is everywhere continuous. The theory may be extended to multiply connected surfaces.

Finally the existence is proved of two regular analytic complex functions $f_a(z)$, $f'_a(z)$ exterior to a given region and two others of the same character $f_j(z)$, $f'_j(z)$ inside the given region, where the following relations

$$f_a(\sigma) = c_1(s)f_j(\rho) + c_2(s)f'_j(s), \quad f'_a(\sigma) = c'_1(s)f_j(\rho) + c'_2(s)f'_j(s)$$

are given between the boundary values of $f_a(z)$ and $f'_a(z)$ at the point $\sigma = \sigma(s)$, of $f_j(z)$ at the point $\rho = \rho(s)$ and of $f'_j(z)$ at the point $s$ of the boundary, where $\sigma$ is a regular differentiable function of $\rho$, and $\rho$ in turn the same kind of a function of $s$. The method of determining these functions is to build up two integral equations in $f_j$ and $f'_j$ similar to those found by Hilbert (*Göttinger Nachrichten*, 1905) for a somewhat simpler problem.

9. Dr. Crathorne considers the problem of determining the minimum of

$$J \equiv \int_1^2 F(y_1', \cdots, y_n'; y_1, \cdots, y_n; x)dx = \text{minimum}$$

under the conditions

$$J_i \equiv \int_1^2 G_i(y_1', \cdots, y_n'; y_1, \cdots, y_n; x)dx = l_i \quad (i = 1, 2, \cdots, r).$$

From the $(2n + r)$-parameter set of solutions of the Euler equations of the problem, it is possible to choose an $n$ parameter set satisfying certain conditions analogous to the condition of transversality in the plane problem. The integral

$$J^* \equiv \int_1^* \{F^* + \sum F^*_{P_i}(y_i' - P_i)\}dx \quad (i = 1, 2, \cdots, n),$$

where $F^* = F + \sum \lambda_i G_i$, is the solution of the Hamilton equation of the problem. Each solution of this equation determines a set $P_1, P_2, \cdots, P_n$ such that the solutions of $y_i' = P_i$ form an $n$-parameter set which is shown to be equivalent to the above $n$-parameter set of solutions of Euler's equations. A "field" is thus established in which $J^*$ is a point function.

10. The paper by Mr. Escott discusses the converse of the theorem of Fermat. If $p$ is any prime and $e$ any integer not divisible by $p$, $e^{p-1} - 1 \equiv 0 \pmod{p}$. According to Ball's Mathematical recreations, the Chinese have a theorem that the condition $2^{p-1} - 1 \equiv 0 \pmod{p}$ is a sufficient condition for a prime number. This theorem has been shown to be untrue by a number of persons. In this paper, which is based on one published in the *Messenger of Mathematics* in March, 1907, Mr. Escott gives a method by which composite numbers can be found containing as many factors as desired and which satisfy the above congruence.

11. In this paper Professor Brenke considers the series

$$\sum_1^\infty (a_n \cos ux + b_n \sin ux),$$

and determines sufficient conditions for convergence, summability (based on the definition of Cesàro) and existence of

derivatives of the function defined by the series. The results are obtained by transformation by a factor

$$\phi(x) = R(\cos \tfrac{1}{2}bx, \sin \tfrac{1}{2}bx),$$

$R$ denoting a rational integral function of the arguments indicated. Application is made to increasing the rate of convergence of trigonometric and other series, and the sums of certain classes of series are obtained in closed form.

12. In his "Tactical memoranda"* Moore defines systems of $k$-ads, $k$-ids, $\underset{\cdot}{k}$-ads, $\underset{\cdot}{k}$-ids. A $k$-ad may be thought of as any one of a symmetric class of $k$-ids in $k$ elements and hence may be called a symmetric $k$-id. A $\underset{\cdot}{k}$-id is some one of the sets of 2-ids obtained from the open linear chain $x_1x_2 \quad x_2x_3 \cdots x_{k-1}x_k$ of 2-ids in $k$ distinct variables, each variable having the range $a_1a_2 \cdots a_k$ independently. Thus a $\underset{\cdot}{k}$-id is what Mr. Schweitzer elsewhere has called an ordered $\underset{\cdot}{k}$-ad. A $k$-id is simply an open linear chain of 2-ids in $k$ elements. A closed linear chain of 2-ids in $k$ elements is a Moore $S\{2, 1k\}$ but not necessarily conversely.

Between the proper $k$-idic, and the symmetric $k$-idic is the alternating $k$-idic standpoint; the latter is considered chiefly in Mr. Schweitzer's paper. A set of connected $k$-ids with the index of connection $(k, p)$, $p = 1, 2, \cdots, k$ is defined; every $k$-id is a set of connected $k$-ids, index $(k, k)$. For example, the 3-ids $mbc$, $amc$, $abm$, have the index (3, 1); the 3-ids $mbn$, $mnc$, $amn$, have the index (3, 2), etc. Corresponding to each type of connection, types of open and closed chains of $k$-ids are defined. A set of alternating $k$-ids with index of connection $(k, 1)$ is reduced formally to chains with index (2, 1), and these chains are also formed by the suitable superposition of linear chains of 2-ids. Mr. Schweitzer has elsewhere defined an $n$-dimensional permutation ($n = 1, 2, 3, \cdots$) and shown that every such permutation contains an $n$-dimensional open chain. It is here shown that every $n$-dimensional chain is reducible to chains with index (2, 1). Finally, the resolution of an alternating set of $k$-ids in $m$ elements into chains of alternating $k$-ids is dis-

* *Amer. Jour. of Math.*, vol. 18, pp. 264–303. The possibility of a connection between the tactical systems here set forth and the above memoir was suggested by Professor Moore. The author takes great pleasure in acknowledging the stimulating influence of Professor Moore and the unusual advantages which have thus been most generously placed at his disposal.

cussed. This leads to infinitudes of finite systems which are important for independence considerations in the author's geometric systems. Particular cases of Moore's $S\{k, l, m\}$ arise frequently and in an interesting synthetic manner.

13. Dr. Birkhoff's paper develops an existence and oscillation theorem for the solutions of a differential equation

$$\frac{d^2u}{dx^2} + q(x, \lambda)u = 0$$

fulfilling self-adjoint boundary conditions at $a$ and $b$. The function $q(x, \lambda)$ increases with the parameter $\lambda$. The methods are similar to those which Professor Bôcher has employed to consider a special case. The paper will be offered to the *Transactions*.

14. The note of Professor Moore will appear in full in an early number of the BULLETIN. The form of analysis in question is that in which one at least of the independent variables enters without direct specification of nature and range of variation.

15. The first paper by Professor Dickson is devoted, in the main, to the reduction, to non-equivalent types, of families of ternary quadratic forms based on two linearly independent forms. The analysis applies to a general field or domain of rationality. The paper has been offered for publication in the *Quarterly Journal of Mathematics*.

16. The paper by Professor Dickson on commutative linear groups is a continuation of the one presented at the last summer meeting (abstract in BULLETIN, November, 1907, pages 61, 62). The complete article has been offered for publication in the *Annals of Mathematics*.

17. In this note Mr. Buchanan states and proves the following theorems: (1) If a sequence of functions, monotonic non-decreasing, converges to a continuous limit function, then the limit function is monotonic non-decreasing and the convergence is uniform. (2) If a sequence of functions $f_n(x)$ $(n = 1, 2, 3, \cdots, n)$ be of finite variation in an interval $ab$ and

convergent to a continuous limit function $f(x)$, also of finite variation in $ab$, and if at every two points $x_1, x_2$, $a \leqq x_1 < x_2 \leqq b$, the limit of the variation of $f_n(x)$ is equal to the variation of $f(x)$, then the convergence is uniform in $ab$.

18. In a paper presented to the Society in April, 1907, Dr. Underhill showed an invariantive normal form of the second variation of

$$J = \int_{t_0}^{t_1} F(x, y, x', y')dt,$$

namely,

$$\delta^2 J = \int_{\alpha_0}^{\alpha_1} \left[ \left( \frac{dv}{d\alpha} \right)^2 - \bar{\kappa}_0 v^2 \right] d\alpha, \text{ where } \alpha = \int_{t_0}^{t} F(x, y, x', y')dt$$

and where $v$ and $\bar{\kappa}_0$ are absolute invariants under extended point and parameter transformations connected with the integrand function of $J$.

The Jacobi equation in this case is

$$\psi(v) \equiv -\bar{\kappa}_0 v - \frac{d^2 v}{d\alpha^2} = 0.$$

Let $v = \phi(v)$ be a solution vanishing for $\alpha = \alpha_0$. In order to draw conclusions as to the next zero of $v = \varphi(\alpha)$, use is made of the equation

$$\frac{d^2 v}{d\alpha^2} + \frac{1}{a^2} v = 0,$$

which has as a solution vanishing at $\alpha = \alpha_0$

$$v = c \sin\left( \frac{\alpha - \alpha_0}{a} \right),$$

and with the next zero at $\alpha = \alpha_0 + \pi a$.

By means of two theorems on differential equations (cf. Darboux, Théorie des surfaces, III, § 629), by which the solutions of the Jacobi equation are compared with those of the last equation, the following theorems may be at once formulated:

I. For the problem of minimizing $J$, if $1/\bar{\kappa}_0$ is positive and less than $a^2$ along the extremal, then the extremal cannot be a minimizing curve in an interval greater than $\pi a$.

II. If $1/\bar{\kappa}_0$ is positive and greater than $b^2$ along the extremal, then the extremal furnishes a weak minimum in an interval at least equal to $\pi b$.

III. In case the problem is that of geodesics on a surface, $\bar{\kappa}_0$ is the gaussian curvature and the above theorems reduce to those already stated by Bonnet, *Comptes rendus*, volume 40, page 1311, also volume 40, page 32.

19. In this preliminary paper Professor Hedrick discusses certain consequences of the Lindelöf definition of a point of condensation (see Borel, Leçons, 1905, Chapter I). Many of the results are not new and a detailed statement is withheld pending further investigation along the lines indicated.

20. The paper of Professor Haskins is a geometric interpretation of L' Hôpital's theorem on the generalized law of the mean. It will be offered to the *Annals of Mathematics* for publication.

21. The motion of a system of tidally disturbing bodies is conditioned by the conservation of the moment of momentum with respect to any plane, and by the fact that the total kinetic and potential energy of the system can only decrease. Professor Moulton's paper develops the consequences of these conditions in the various cases which can arise. Applications are made chiefly to the earth-moon system, and to double stars. Among other things it is shown that tidal friction can not have been an important factor in developing the distances between the stars in binary systems.

22. Professor Hathaway considers a reduced problem for arbitrary motion, analogous to Lagrange's reduced problem for gravitational motion. The reduced elements are the three mutual distances and three relative velocities of the bodies (in magnitude only). When these are given, we can determine algebraically the angle between the plane of the bodies and the plane parallel to their relative velocities, the angular velocities of these planes, their inclinations to the axis of moment of momentum, etc. The complete solution of the problem is by linear differential equations in terms of the reduced elements whose complete solutions are arbitrary constant rotations of a particular solution. When one set of these differential equations is

solved, the other set may be reduced to quadratures. If the axis of moment of momentum is fixed, the complete solution is by quadrature, thus paralleling the gravitational problem.

It also appears that Lagrange's investigations in the problem of three bodies are generally independent of the law of gravitation. For example, those motions in which the triangle of the bodies is always similar to itself require that triangle to be either equilateral or with collinear vertices, and the sides to revolve in a fixed plane through the center of gravity. It is shown here that the angular velocity of the triangle varies inversely as the square of a side and so is constant for rigid configurations. Also that no other solution than the above is possible for constantly collinear bodies whose co-line is not a fixed line.

Lagrange's biquadratic equation for $\rho$ holds also for arbitrary motion, and his differential equation for the same in any gravitational motion has the roots of that biquadratic for particular, and not singular, solutions.

H. E. SLAUGHT,
*Secretary of the Section.*

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## JOINT MEETINGS OF MATHEMATICIANS AND ENGINEERS AT THE UNIVERSITY OF CHICAGO.

A SERIES of meetings of mathematicians and engineers was held at the University of Chicago, December 30–31, 1907, under the auspices of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY and conjointly with Sections A (mathematics and astronomy) and D (mechanical science and engineering) of the American Association for the Advancement of Science.

The invitation to join in the discussion of the teaching of mathematics to students of engineering had been widely distributed among those engaged in the practice of engineering as well as among professors in technical schools. The attendance was large and representative, including one hundred men especially interested on the mathematical side and fifty on the engineering side. Among the institutions represented were the State Universities of Illinois, Indiana, Iowa, Kansas, Minnesota, Missouri, Nebraska, Ohio, Pennsylvania, Vermont,

West Virginia and Wisconsin; the following technical schools: Armour Institute of Technology, Bradley Polytechnic Institute, Case School of Applied Science, Lewis Institute of Chicago, Massachusetts Institute of Technology, Michigan College of Mines, Michigan Agricultural College, Purdue University, Rose Polytechnic Institute, Stevens Institute of Technology, and the Agricultural and Mechanical College of Texas; and the following institutions: Brown University, The Carnegie Institution of Washington, University of Chicago, University of Cincinnati, Colgate University, Harvard University, Knox College, Miami University, James Millikin University, Kalamazoo College, New Hampshire College, Oberlin College, Ohio Wesleyan University, University of the South, Union College, Washington University, Wheaton College, and the United States Weather Bureau.

The promotion of acquaintance and good fellowship was an important feature of the meetings, which was fostered by the admirable arrangements provided by the University of Chicago and especially by the subscription dinner attended by one hundred engineers and mathematicians. The speakers at the dinner, introduced by Professor E. B. Van Vleck, Chairman of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY, were Professor Calvin M. Woodward, Dean of the School of Engineering of Washington University, St. Louis, Mo., Mr. Charles F. Scott, Chief Consulting Engineer of the Westinghouse Electric and Manufacturing Company, Pittsburgh, Pa., Professor George A. Swain, Massachusetts Institute of Technology, Boston, Mass., and Professor Edward V. Huntington, Harvard University, Cambridge, Mass.

The presentation of the papers, covering two half days, led to an enthusiastic discussion which extended to a third session and which culminated in the appointment of a committee to take into consideration the whole question of the mathematical curriculum in technical schools and in technical departments of colleges and universities, and to report to a joint meeting of engineers and mathematicians to be held in 1909 in connection with the annual gathering of the Society for the promotion of engineering education. The selection of this important committee was entrusted to Professor E. V. Huntington of Harvard University, Professor Gardner S. Williams of the University of Michigan, and Professor E. J. Townsend of the University of Illinois, who themselves are to constitute the nucleus of

the committee and who were given full power to select the remaining members, choose a chairman, and determine the scope of the investigation to be entered upon, it being understood that the committee shall number not less than fifteen and shall be widely representative both geographically and as to all the institutions and interests involved.

At the first session four papers were presented as follows:

"The present status of mathematical instruction for engineers in American schools," Professor E. J. TOWNSEND, University of Illinois.

"The present status of mathematical instruction for engineers in foreign schools," Professor ALEXANDER ZIWET, University of Michigan.

"The point of view in teaching engineering mathematics," President R. S. WOODWARD, The Carnegie Institution of Washington.

"The scope and spirit of mathematical instruction for engineers," Mr. CHARLES F. SCOTT, consulting engineer, Westinghouse Electric and Manufacturing Company.

The second session consisted of a symposium on the topic: "What is needed in the teaching of mathematics to students of engineering?" The subdivisions of the topic were:

(*a*) What range of subjects? (*b*) To what extent in the various subjects? (*c*) By what methods of presentation? (*d*) What should be the chief aims?

The speakers, who represented three phases of the subject, namely, (1) the standpoint of the practicing engineer, (2) the standpoint of the professor of engineering, (3) the standpoint of the professor of mathematics in the engineering school, were as follows:

RALPH MODJESKI, consulting civil engineer, Chicago, Ill.

CHARLES S. SLICHTER, professor of applied mathematics and consulting engineer, University of Wisconsin.

GARDNER S. WILLIAMS, professor of civil hydraulic and sanitary engineering and consulting engineer, University of Michigan.

FREDERICK S. WOODS, professor of mathematics, Massachusetts Institute of Technology, Boston, Mass.

GEORGE F. SWAIN, professor of civil engineering, Massachusetts Institute of Technology, Boston, Mass.*

* Professor Swain, on short notice, supplied the place of Mr. W. L. Abbott, chief operating engineer of the Chicago Edison Company, who was unexpectedly unable to be present.

ARTHUR N. TALBOT, professor of municipal and sanitary engineering in charge of theoretical and applied mechanics, University of Illinois.

FRED. W. MCNAIR, president of the Michigan College of Mines, Houghton, Mich.

J. A. L. WADDELL, consulting bridge engineer, Kansas City, Mo., whose paper was read by Professor O. D. Kellogg.

The papers will be printed in full during the year 1908 in *Science* and will thus reach all who are members of the American association for the advancement of science.

Abstracts of the papers and a list of those who contributed to the general discussion are given below:

Professor Townsend considers the rapidly increasing demand for trained men in the field of engineering and the remarkable growth in recent years of the technical schools, both in number and in equipment and efficiency, and shows that the time is indeed ripe for serious consideration, on the part of both mathematicians and engineers, of all phases of the question of mathematical training for engineering students. He has selected for comparison seventeen institutions in this country where engineering training is either an important or the exclusive feature and he considers three questions with respect to the mathematical training of engineering students in these institutions, as representative of the status in the country at large; namely, (1) the entrance requirements, (2) the requirements for graduation, (3) the qualifications of the instructional force. Under the first and second of these heads Professor Townsend exhibits a comparative tabular statement and draws a number of incisive conclusions both as to the facts and as to the tendencies, all of which may well be pondered carefully by everyone interested in the industrial and scientific development in America. These data and preliminary observations will be of prime importance to the committee of fifteen who are to investigate and report upon the whole question of the mathematical curriculum in the technical schools. Under the third head also Professor Townsend has clearly set forth the difficulties and the demands of the situation, and it will be the province of the general committee to report also upon these matters. It is greatly to be desired that all mathematicians may not only read this paper, as it will appear in full in *Science*, but that many suggestions may be communicated to the committee through Professor Town-

send. To this end the following questions suggested by his paper are proposed:

### I. Entrance Requirements.

1. Is a greater uniformity of entrance requirements desirable?
2. Most colleges in the middle west admit on certificate from accredited schools. In addition to this should engineering students be required to pass an entrance examination in algebra, with the understanding that if they fail to make satisfactory grade, more than the usual amount of work must be done to secure credit in college algebra?
3. Should a knowledge of logarithms and the plotting of simple algebraic curves be added to the entrance requirements?
4. Should the standard of admission be raised so as to include trigonometry and college algebra?
5. Should the requirements be made to cover less ground and be correspondingly intensified?
6. Should more attention be paid to analytic and formal work, particularly in arithmetic and algebra?
7. Should a year of work in mathematics and science of college grade be required for entrance, or should the college course be extended to five years?

### II. Requirements for Graduation.

1. What should be the relative length of time spent on algebra, trigonometry, analytical geometry, and calculus?
2. Which should precede, algebra or trigonometry?
3. What topics should be particularly emphasized in college algebra? in calculus?
4. How far ought the instruction of the first two years' work in mathematics to be made "practical"; how far should we insist upon rigorous demonstrations of principles taught?
5. Should students in one line of engineering, say civil engineering, be given problems of a different nature than those given to students in other lines, say mechanical or electrical engineering?
6. Should differential equations and least squares be required subjects in any engineering course? If so, how extensive should these courses be?
7. Should we have a separate course on "applications" having for its purpose the cultivation of ability for rapid com-

putation and the use for engineering work of such instruments as the slide rule, planimeter, integraph, computing machines, etc.?

8. What opportunity for the study of pure mathematics should be given the engineering student beyond the usual course in calculus? What courses might be made elective in the junior or senior years?

9. Should a first course in mechanics be given to engineering students in the freshman year and before the student has had calculus?

### III. Administrative Questions.

1. What qualifications should we insist upon for the instructor of engineering students in mathematics?

2. How much elementary mechanics should be taught in connection with the calculus? Should this elementary mechanics be taught by the mathematical department?

3. Should the work in descriptive geometry be made more mathematical in treatment? Should it be taught by the mathematical department?

4. What can be done in general to bring about a closer relation between the teachers of mathematics and the teachers of engineering?

Professor Ziwet refers to the fact that remarkable and characteristic developments have taken place in England and Scotland, and that France represents probably the highest standard of any country with respect to mathematical training for engineering students, requiring, for instance, for admission to the Ecole polytechnique almost as much mathematics as would be required for graduation from our technical colleges. He considers, however, in some detail only the technical curricula of the German schools with which he is personally familiar, and reaches the general conclusion that while the average German engineer may ultimately know no more mathematics than the average American engineer, yet an able German student in his technische Hochschule, or engineering university, can get a more thorough scientific equipment than an equally able American student in his alma mater. One important advantage in the training of the German student is the fact that his preparatory mathematics (including arithmetic) is distributed systematically and continuously over a period of nine years, whereas in American schools there is much discontinuity and lack of homogeneity

all along the line from the grammar school through the high school and freshman and sophomore years of the college course. Professor Ziwet gives an excellent resumé of the three phases of reform in the teaching of mathematics which are now commanding attention; namely, one which was originated by the German association of engineers, one which was promulgated by the university authorities, and a third which was proposed in connection with the committee of the German association of Naturforscher and Aertzte. From all these considerations Professor Ziwet draws thoughtful conclusions as to the situation in America and suggests some timely notes of warning.

President Woodward maintains that the fundamental ideas which belong to mathematics and mathematical physics and which form an essential part of engineering training are more inherently difficult than we commonly suppose, and that failure to recognize this in our teaching is the most glaring fault. We marvel at the ease with which the student forgets or fails to comprehend what we try to teach him. We must expect this and must therefore patiently continue to insist upon attention to the fundamental and elementary notions, and drive them home by unremitting practice in computational applications. President Woodward would have most of our text-books remodelled so as to present first the plain statement of facts and much later the theoretical and abstruse considerations. He takes pride in the high development in this country of pure mathematics and of experimental physics, but deplores the conspicuous lack of men highly trained in both mathematics and physics such as the French have produced — who combine both the theoretical and the practical and who therefore are prepared to advance the standards of scientific attainment in engineering lines. He attributes the dearth of such men in the theoretical field and in the teaching profession to the alluring remunerations held out to those who enter the commercial side of engineering work. President Woodward finds great hope for the future in this co-operation of mathematicians and engineers which has been inaugurated in these joint meetings and which bids fair to be perpetuated and extended through the mutually cordial sentiment here expressed and especially through the forces here put into operation.

Mr. Scott's paper presents many points for serious consideration to the teacher of pure mathematics. He appreciates fully

the theoretical point of view with respect to the training of an engineer, yet he is in a position to realize how intensely practical the average engineer must be and how little use he has for any mathematics which he cannot instantly translate into the terms of his work-a-day environment. At the outset he discusses three questions: What are the uses to which an engineer may apply mathematics? What kind of mathematics does he need? What skill does he require in the use of mathematics? He then proceeds to discuss what mathematical subject matter should be covered by a student of engineering and how it should be taught. In this connection he lays great stress upon the training of the judgment and the development of the mathematical *sense*. It is one thing to know higher mathematics, or mathematics of any kind, and quite another thing to know how to use this knowledge in the practical world in bringing things to pass. "The ability to state a problem, to recognize the elements which enter into it, to see the whole problem, not overlooking some important factor, to use good judgment as to the reliability of the data involved, to be able to interpret the result, to recognize its physical significance — such powers as these are of higher order and of greater significance to the engineer than the ability to take a stated problem and work out the answer. It may be said that this is not strictly mathematics, but it is just the judgment and insight which make mathematics really useful and effective." Mr. Scott concludes with a paragraph on the relation of education to the affairs of life which every teacher of mathematics may well ponder.

Mr. Modjeski speaks from the standpoint both of a man highly trained in the Ecole des Ponts et Chaussées of France and also of a highly successful consulting bridge engineer of Chicago. While he would in no wise depreciate the stimulating effect of the mental discipline derived from a broad general course in the higher mathematics, he would call attention to the present agitation in France and Germany, especially in France, looking toward the curtailment of the current mathematical program in the engineering schools, on the ground that it is unnecessarily extensive, considerably more so than in this country. Mr. Modjeski would raise the question as to whether we should not specialize the mathematical courses and select for each branch of engineering those particularly adapted to training in that line. For instance, a railroad engineer who aspires to

become a railroad official requires less knowledge of the calculus than the electrical or bridge engineer, but on the other hand he requires a greater knowledge of geology and common law than either of these. The bridge engineer demands a higher mathematical training than any other branch of the profession, and yet such a course as the theory of differential equations will be of little if any use to him and his time might better be given to such a course as, for instance, the methods of least work, which no bridge engineer should neglect. Mr. Modjeski would advocate the replacing of the abstract and meaningless problems of the elementary text-books by problems of a more practical and interesting nature, and above all he would insist upon a training in mathematical sense, a development of the habit of mathematical thinking to such a degree that the prospective engineer shall be able to apply the fundamental principles as one to whom they have become his second nature.

Professor Slichter points out that, whereas engineering technology was founded chiefly on practice rather than on theoretical principles, it has now grown to be an investigative science of such proportions that few scientific productions now excel the engineering and technological treatises which come from the press. The need of the engineer is not so much that his mathematics shall be taught to him in some different way as that he shall have *more* mathematics. The engineer does not ask for better instruments or more difficult projects or more capital to conduct operations, but for more knowledge. It is a hopeful sign that many engineering institutions are now fostering some phase of investigative work. The chief requirement of the mathematical curriculum is that it shall be compact and stable. There is very little time for the engineering student to dally in mathematics with whims and fads and new schemes. Mathematics is to the engineer not merely a tool, but like physics and chemistry, it is to him a basal science, and should be made to appeal to his interest and his experience. He will forget his mathematics, to be sure, but he will forget his hydraulics also, and the one as easily as the other, if they are not both intrenched by strong ties of interest. Professor Slichter would plead for more opportunity for electives in mathematics in engineering courses but would insist that, so far as the teaching is concerned, any shortcomings in the engineering profession are quite as likely due to faults in the technological courses themselves as in the mathematical courses.

Professor Williams recalls the fact that engineering a half century ago was merely an art, but that it has now risen to the standing of a science as well as an art, and that from this wider point of view only does mathematics have a place in the training of the engineer. Engineering as a science is hampered by the trade school which naturally views it as an art only. The products of the trade school have little use for the mathematics of engineering. Both the trade school and the highly specialized technical school have their place. While the vast majority of those connected with engineering work possess and use a very limited mathematical training, yet we may well consider the mathematical equipment demanded by the man who is to climb to the top of his profession. Professor Williams maintains that what the trained engineer needs is not more formulas but a better understanding of principles and a stronger development of the reasoning power, so that he may indeed be able to think mathematically. To this end the mathematical teaching of the grammar school needs overhauling and next that of the high school, the implication being that too much formal and abstract work is now required, and too little development of independent thinking through the solution and interpretation of practical problems. Mathematics is a tool but it is to be used with intelligence and not in the blind following of rules; it is, so to speak, a living instrument which must respond like an intelligent servant. As to the question of increasing the mathematical requirements for engineering students, there is danger that this may work a disadvantage if this is done at the sacrifice of physics, chemistry, and the other sciences. Elective courses in mathematics should be provided in the engineering course but not till the fourth year, as few students are competent to elect intelligently before that time.

Professor Woods refers to the fact that while the best technological schools offer ample elective work in such mathematical courses as advanced calculus, least squares, differential equations of physics and mechanics, etc., yet their chief concern is not the development of mathematicians as such, but the training of engineers who shall be able to use mathematics in both practical and theoretical work. He has no patience with the professors of engineering who undertake to direct the work of mathematics in an engineering school. He believes that the teachers of mathematics must be mathematicians and that, while

they should constantly connect their work with the concrete side, yet they must teach mathematics as mathematics. He believes that the teaching of mathematics, like all other teaching, is capable of improvement and that great improvement has been made and is now in progress, but he is not willing to admit that the teaching in this line is worse, or has been worse, than in other lines, as some engineers would seem to imply. The mathematical department delivers its product to the engineering department of the technical school, there to be subjected to the searching critical tests of further years of training, while the engineering department delivers its product to the outside world there to be swallowed up in the multitude, whose criticisms are aimed at the man himself rather than at those who have been his teachers. Professor Woods gave a clear exposition of the fundamental steps needed in the successful teaching of mathematics in its relation to its concrete applications and elicited much interest in the modification of the curriculum now being put into effect at the Massachusetts Institute of Technology.

Professor Swain is doubtful as to the worth of mathematical study other than as a means to an end — the development of a tool with which to do something. He is disposed to assign low value to mathematics as mere training in mental power, in comparison with some other branches of science. He would criticize the present teaching of mathematics as too ideal and abstract, looking for logical and theoretical results which conform to the demands of rigid demonstration regardless of common sense, clear observation, and good judgment. From the standpoint of the engineer, for whom mathematics is simply a useful tool, and whose training, therefore, should lead to power in handling the instrument, Professor Swain would point out that the present unsatisfactory results may be due in part to the lack of consecutiveness in the mathematical curriculum, especially in the latter part of the secondary course, which permits a lapse of from one to three years in the study of algebra, and also at other points in the curriculum allows the presentation of subjects in too isolated and incoherent fashion. He also asserts that too much attention is given to pure analysis and not enough to geometry. To the engineer, he says, geometry is all important. The geometric demonstration is grasped by the mind and comprehended step by step, while analysis is like a machine which transforms the data into the desired con-

clusion by some occult process which the mind does not fully grasp. He would also revive the study of mental arithmetic and lay far more stress on all the mental processes of mathematical work. Finally he would discredit the lecture system of teaching mathematics, and would insist upon the Socratic method of question and cross-question in all class work, and would have the teaching of mathematics done by engineers or at least by men acquainted with the engineering applications of mathematics.

Professor Talbot discusses the mathematical training of engineering students from three points of view; namely, theory, practice, and philosophy or interpretation. His contention is that a proper relative proportion of emphasis on these three phases is of more importance than the content or extent of the curriculum, provided the principles covered are really understood and intelligently interpreted and applied. He emphasizes the fact that we must deal with the average student and that it is only the exceptional student who is the mathematical genius, hence the methods of presentation must be such as to encourage and develop the great middle class of engineering students. Professor Talbot believes that, while formerly we may have carried theoretical and demonstrational processes to an extreme, we are now likely to swing to the other extreme, and let the student conclude that the facts are of chief importance and the proof of principles involves useless effort. He would insist that practice in analysis and formal demonstration is illuminating and developing to the mind, that even the old time mental arithmetic had its great educational value. Likewise, practice and drill work have their uses and abuses. To the average student the working of problems is illuminating and educative. Mathematics is a tool and the engineering student must acquire facility in its use and this demands drill and repetition. But the students who think that to accept facts and work problems is sufficient, and the instructors who think that illustration and practice alone constitute mathematical training or that mere laboratory methods suffice, are greatly mistaken. Again, there must be a direct connection between the theory and the philosophy of the subject to make the practice side serve its proper purpose. Professor Talbot would discourage the lecture method for engineering students and would select with great care the more advanced and complex courses which are offered, espe-

cially those involving concepts lying beyond the student's experience.

President McNair considers not so much the ideals which we might set up for the mathematical curricula of the engineering schools, as the nature of the actual demands which are made upon the graduates whom we turn out. The problem of the school is to meet the demand for men who can do something, men who can think in a logical and common sense way, men who are ready to accomplish results from the outset. The demand is not for highly trained mathematicians (however much we may think it is) but for men who can use a little mathematics and use it effectually as a tool. The danger is that the mathematical requirement may be too high. The mathematical concepts are difficult and it may be that we are expecting too much, too varied, and extensive attainments. The great thing is to gain ability to think correctly and logically about things, and in this respect the teacher of mathematics and the teacher of engineering have the same task before them. There is no better place than the mathematical class-room to develop logical thinking, but it should be done in connection with well selected concrete problems, rather than in the realm of abstract and theoretical considerations. The man who can take hold of a problem, analyze the data, put together the facts and reason logically to a sound conclusion, is already on the road to a successful career, and the teacher who is helping his students to attain such power is fulfilling his mission to the student who is to prepare for engineering.

Mr. Waddell refers to the teaching of mathematics to engineering students twenty years ago as sufficiently strenuous but far from satisfactory and finds little indication of radical improvement at the present time. He holds that the engineering student in his pure mathematical classes is not taught what the equations employed really mean, but that much of his work is a juggling with quantities to produce certain results, while it is left to the teacher of rational mechanics to bring out the reality of mathematics. He would advocate greater emphasis upon the interpretation of symbols used, the constant resort to graphical methods, expecially in analytics, the concrete application of all principles developed, especially in the calculus, a greater attention to the subject of descriptive geometry early in

the engineering curriculum, and a thorough mastery of mechanics, whose foundation is mathematics and which underlies the superstructure of engineering. Mr. Waddell is strongly opposed to the lecture system of teaching mathematics to engineering students. Aside from introductory and concluding lectures, together with informal talks from time to time on the general trend of the subject, he would insist on the midnight oil and the damp towel as the necessary accessories for obtaining a real and comprehensive grasp of the subject for the average student. As to the instructional staff in engineering schools he would insist that the teachers of mathematics should also be engineers and should have a practical acquaintance with both rational and practical mechanics. And finally as to the extent of mathematical study for an engineer, while it is true that the actual use of analytic geometry, calculus, least squares, etc., rarely occurs in the practice of most engineers, yet the engineer's grasp of technical work depends upon his knowledge of these subjects and hence his mathematical foundation must be strong and substantial.

The general discussion was supported by Mr. C. F. Scott, Pittsburg, Pa., Dean C. M. Woodward, Washington University, Professor B. F. Groat, School of Mines, University of Minnesota, Professor S. M. Barton, University of the South, President C. S. Howe, Case School of Applied Science, Professor C. A. Waldo, Purdue University, Professor C. B Williams, Kalamazoo College, Mr. J. B. Webb, consulting engineer, Hoboken, N. J., Dean H. T. Eddy, College of Engineering, University of Minnesota, Professor D. F. Campbell, Armour Institute of Technology, Professor A. E. Haynes, College of Engineering, University of Minnesota, Professor E. W. Davis, University of Nebraska, Professor A. S. Hathaway, Rose Polytechnic Institute, and Professor E. V. Huntington, Harvard University.

Abstracts of these shorter addresses are not immediately available, but doubtless some of the more important utterances may be printed in connection with the formal papers in *Science*.

H. E. SLAUGHT,
*Secretary of the Chicago Section.*

# THE FIFTY-EIGHTH MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE fifty-eighth meeting of the American Association for the Advancement of Science was held at the University of Chicago during the convocation week December 30, 1907, to January 4, 1908. The president of the meeting was Professor E. L Nichols, Cornell University. The address of the retiring president, Dr. W. H. Welch, entitled "The interdependence of medicine and other sciences of nature," was given in the Leon Mandel Assembly Hall on the evening of the opening day.

The meetings of Section A were of unusual interest in view of enthusiastic joint sessions with Section D and the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY to consider the present status and means of improvement of the teaching of engineering students of elementary mathematics, and also to listen to the address of the retiring vice-president, Professor Edward Kasner, of Columbia University, entitled "Geometry and mechanics." The officers of the section were: vice-president, E. O. Lovett; secretary, G. A. Miller; councilor, G. B. Halsted; member of the general committee, F. R. Moulton; sectional committee, Edward Kasner, E. B. Frost, E. O. Lovett, Harris Hancock, F. R. Moulton, E. W. Brown, and G. A. Miller. The following program was presented at the separate meetings of Section A:

(1) Professor E. B. FROST: "Observations with the Bruce spectrograph."

(2) Professor E. B. FROST: "Comments on the Zeiss stereocomparator and the spectrocomparator belonging to the Yerkes observatory."

(3) Professor E. E. BARNARD: "Photographic phenomena of comet *d*, 1907 (Daniel)."

(4) Professor E. E. BARNARD: "On a great bed of nebulosity in Sagittarius, photographed with the Bruce telescope of the Yerkes observatory."

(5) Professor E. O. LOVETT: "Note on the problem of three bodies."

(6) Professor JOEL STEBBINS: "The light curve of Delta Cephei."

(7) Professor JOEL STEBBINS and Mr. F. C. BROWN: "An application of the selenium cell to astronomical photometry."

(8) Professor E. E. BARNARD: "Observations and explanation of the phenomena seen at the disappearances of the rings of Saturn."

(9) Professor E. B. FROST and Mr. PHILIP FOX: "On the twenty-foot horizontal solar spectrograph of the Yerkes observatory."

(10) Professor KURT LAVES: "A graphic method for the determination of the orbit of a spectroscopic binary."

(11) Professor KURT LAVES: "New tables for the time of sight correction of the earth's orbital motion."

(12) Professor W. D. CAIRNS: "A generalized theory of integral equations."

(13) Professor G. B. HALSTED: "On the theory of order, static and nascent."

(14) Dr. F. W. REED: "Singular points in the approximate development of the perturbative function."

(15) Professor JOHN EIESLAND: "On a certain class of algebraic translation surfaces."

(16) Dr. ARTHUR RANUM: "Matrices not belonging to groups."

(17) Professor C. H. BECKETT: "A note on interest on reserve from items computed for the uniform report blank."

(18) Dr. A. E. YOUNG: "On asymptotic isothermic surfaces."

(19) Professor G. A. MILLER: "Some questionable terms and definitions used in elementary mathematics."

(20) Messrs. J. A. PARKHURST and F. C. JORDAN: "The photographic determination of star colors and their relation to spectral type."

(21) Mr. PHILIP FOX: "On the detection of the eruptive prominences on the solar disk."

(22) Mr. PHILIP FOX: "An investigation of the 40-inch objective at the Yerkes observatory."

(23) Mr. R. J. WALLACE: "The function of a color filter and of certain plates in astronomical photography."

(24) Professor G. E. HALE: "The vertical coelostat or 'tower' telescope of the Mt. Wilson solar observatory."

(25) Professor G. E. HALE and Mr. W. S. ADAMS: "Preliminary results of a comparative study of the spectra of the limb and the center of the sun."

(26) Professor FRANK SCHLESINGER: "A simple method for reducing spectrograms."

(27) Professor F. R. MOULTON: "On the probability of the near approach of two stars and on relative problems in the sidereal universe."

In the absence of their authors the papers by Professors Cairns, Halsted and Lovett and Dr. Ranum were read by title; that by Professor Schlesinger was presented by Mr. Fox. All the other papers in the above list were read by their authors. The following abstracts of those which were of mathematical interest bear numbers corresponding to those of the titles in this list. The abstracts of the others appeared in the report of this meeting published in *Science*, January 31, 1908.

5. Professor Lovett constructs a problem of three bodies possessing exact transcendental solutions defined by finite equations; these solutions reduce to the lagrangian solutions of the classic problem of three bodies when the ideal problem assumes the newtonian form. The paper is a part of an extended memoir which is not yet ready for publication.

12. By a generalization of the Hilbert theory of a quadratic form in an infinite number of variables, Professor Cairns gives a complete treatment for the integral equation

$$f(s) = \phi(s) - \lambda \int_a^b K(s, t)\phi(t)dt + x'g(s)$$

with the auxiliary equation

$$\int_a^b g(t)\phi(t)dt = c,$$

$f(s)$, $g(s)$, and $c$ being given, $K(s, t)$ a generalized Green's function, and $\phi(s)$ the required function.

Application is made to the differential equation

$$p(x)\frac{d^2u}{dx^2} + \frac{dp}{dx}\frac{du}{dx} + q(x)u + \lambda' g(x) + \phi(x) = 0$$

with the auxiliary equation

$$\int_a^b u(x)g(x)dx = 0,$$

to a generalized Fourier development, and to the isoperimetric problem

$$\int_a^b F(y', y, x)dx = \text{minimum},$$

given

$$\int_a^b G_i(y', y, x)dx = l_i \quad (i = 1, 2, \cdots, m).$$

13. The paper by Professor Halsted investigates the essence of inherent order, its foundation and genesis, and considers the uses of inherently ordered series in the attribution of factitious order to primarily unordered assemblages. It gives the solution of problems in arrangement and investigates the betweenness relations, linear and of more than one dimension. It gives applications to the theory of teaching, to arithmetic, and to geometry.

14. The method of approximation developed by Poincaré for obtaining the terms of higher order of the perturbative function depends uniquely upon certain singularities of this function. In the general case the singular points are given by algebraic equations of a high degree. The discussion relative to the actual admissibility of these points is often very delicate, but a criterion once established leads to the solution of the general case by extension of the results found by Poincaré, Hamy, Feraud, and Coculesco in certain restricted cases. With suitable numerical assumptions the case of small eccentricities and a small inclination of the orbits was carried out in detail by Dr. Reed, and the results to be found by varying the elements are indicated. The method is especially applicable to the problem of finding the values of the coefficients with small divisions in the perturbations of the small planets by Jupiter.

15. Professor Eiesland's paper contains a treatment of all the types of translation surfaces that are determined by a unicursal quartic in the plane at infinity. The most important result obtained is this: To a unicursal quartic with three real double points correspond $\infty^3$ types of translation surfaces of the form

$$(1)\quad A + Be^X + Ce^Y + De^Z + Ee^{X+Z} + Fe^{X+Y} + Ge^{Z+Y} + He^{X+Y+Z} = 0$$

where the coefficients satisfy the following identical relation:

$$EGAF = HDCB.$$

By a logarithmic transformation

$$x = \log X, \quad y = \log Y, \quad z = \log Z$$

this surface is converted into the cubic surface

(2) $A + Bx + Cy + Dz + Exz + Fxy + Gzy + Hxyz = 0$

which is the exact analog of the surface

$$Ax + By + Cz + Dxy + Exz + Fzy = 0$$

obtained by Lie for the case of a degenerate quartic consisting of two conics.* It is the intention of the writer to study the surfaces (2) more in detail. The paper will be offered to the *American Journal of Mathematics* for publication.

16. All ordinary $n$-ary matrices (linear substitutions with non-zero determinants) form a group, and some singular matrices (with zero determinants) also belong to groups. If $S$ is a singular matrix not belonging to any group, then if the number of zero roots of its characteristic equation is $s$, there always exists a positive integer $m \leqq s$ such that $S^m$ belongs to some group. If $m$ is the lowest such integer, then among the invariant factors of the characteristic determinant of $S$ that correspond to zero roots there is at least one of order $m$ and none of higher order. Dr. Ranum also showed that if $S$ is of rank $r$, it is reducible to an ordinary $(n - s)$-ary partial matrix and $n - r$ singular, irreducible partial matrices, each of which is nilpotent, *i. e.*, has a power (exponent $\leqq m$) equal to zero.

17. By an agreement among the insurance commissioners and supervisory departments of a majority of the States of the Union, a detailed and comprehensive report of the year's business is required of life insurance companies on a form known as the union form report blank. In the "profit or loss" account of this sheet there is an item "interest required to maintain reserve." The net income from investments is exhibited and from these items the "profit or loss" from interest earnings is shown. There is a discrepancy between the mathematical theory involved in the fundamental calculations, and the policy contract itself which leaves some latitude and consequent confusion. The purpose of Professor Beckett's paper is to set forth this problem and find a solution that can be checked from the other

*See Lie-Scheffers, Berührungstransformationen, vol. 1, pp. 367 and 410.

items required to be computed and exhibited in the report, in the general interest of a fair understanding and uniformity. All life insurance calculations are based upon the theory that premiums are paid at the beginning of the insurance year, and death claims paid at the end of the year. All modern life insurance contracts provide that payment shall be made upon satisfactory proofs of death, or in a few cases within 60 days thereafter.

The death strain or "mortality cost" in any case is the amount at risk multiplied by the probability of dying, *i. e.*, $C_{x+m}(S - V_{x+m})q_{x+m}$. The mean reserve which appears in the report blank is $\frac{1}{2}(P_x + V_{x+n}V_{x+n+1}) = m_{x+n+\frac{1}{2}}$. The net cash value of the policy at the end of any year, or terminal reserve, is the fund which together with the net premiums to be received in the future is the exact mathematical equivalent of the obligation incurred by the company to pay the contract.

In any individual case, interest $3\frac{1}{2}$ %,

$$1.035\,V + P_x(1.035) = V_{x+n+1} + C_{x+n},$$

$$M_{x+n+\frac{1}{2}} = \frac{V_{x+n} + P_x + V_{x+n+1}}{2}, \quad V_{x+n+1} = 2M_{x+n+\frac{1}{2}} - V_{x+n} - P_x,$$

$$V_{x+n} = \frac{2M_{x+n+\frac{1}{2}} - V_{x+n} - P_x + C_{x+n+1} - P_x(1.035)}{1.035},$$

$$V_{x+n} = \frac{M_{x+n+\frac{1}{2}} + \frac{1}{2}C_{x+n+1} - P_x(1.0175)}{1.0175}.$$

Let

$$V_{x+n} + P_x = R, \quad R = \frac{M_{x+n+\frac{1}{2}} + \frac{1}{2}C_{x+n+1}}{1.0175}.$$

Interest for the first half year

$$R(.0175) = \frac{.0175}{1.0175}\,[M_{x+n+\frac{1}{2}} + \tfrac{1}{2}C_{x+n+1}].$$

Let $V_{x+n-1} + P_x = K$.

Interest for the second half year

$$K(.0175) = \frac{.0175}{1.0175}\,[M_{x+n+\frac{1}{2}} + \tfrac{1}{2}C_{x+n+2}].$$

If from these sums there be deducted $3\frac{1}{2}$ per cent. of the mean net deferred premiums, the result gives the interest required to maintain reserve in terms of other items which are reported in the blank. This is a close check and true in theory, but not quite true in actual practice and contract. In case a death claim falls in the early part of the insurance year on a contract that is young, the loss is conspicuous. The only way to make a perfect agreement between theory and contract would be to recompute all net premiums and all other tables involved in the calculations. Upon the new assumption that claims are paid immediately, this is possible but not practical, as it would cause great confusion, and since it could not be retroactive, it would take a long period of years to have all forms in conformity to this practice. In the meantime great confusion would prevail. Should the company earn only this guaranteed rate of interest upon net premiums and reserves, there would be a deficiency of approximately .0035 per cent. on a mixed business of paid-for insurance, endowment, and limited payment forms.

Claims cannot be paid actually at the moment of death. Statistics show that the actual necessary lapse of time averages one month and that on an average the loss of interest upon experienced mortality funds paid out is five months. The best practical adjustment would then be to discount this fund upon this five-month basis and require the company to make good the deficiency out of excess earnings over the rate assumed in the contract.

18. Surfaces characterized by having isothermal asymptotic lines and isothermal lines of curvature Dr. Young has called "asymptotic-isothermic," and in a paper read before the Chicago section of the AMERICAN MATHEMATICAL SOCIETY, March 30, 1907, he outlined a general method for the determination of these surfaces and discussed a particular solution of the problem which led to certain surfaces whose first fundamental forms, when referred to lines of curvature, come under the more general form

$$\overline{ds}^2 = \frac{(u+v)^k}{(u-v)^l}\left(\frac{du^2}{U} + \frac{dv^2}{V}\right),$$

where $k$ and $l$ are constants, and $U$ and $V$ are functions of $u$ and $v$ respectively. He continues the discussion in this paper

and determines all surfaces which have the above expression for their first fundamental form when referred to lines of curvature, and also all isothermic surfaces which have the same spherical representation of their lines of curvature as these. All of the new surfaces are asymptotic-isothermic, with the exception of one class composed of an infinity of surfaces.

19. The term division has two distinct meanings in elementary mathematics.* According to one of these, it implies the operation of finding an integral quotient and an integral remainder, while according to the other it implies the finding of a number which multiplied into the divisor produces the dividend. While only the latter is the inverse of multiplication, yet it is customary to speak of division as an inverse operation without specifying which of the two commonly accepted definitions of the term is meant. A very common definition of multiplication is the performing upon the multiplicand the same operation as that which is performed upon unity to get the multiplier. The vagueness of this definition follows directly from the fact that 4 may be obtained by doubling unity and squaring the result, yet multiplying by 4 does not generally mean doubling the multiplicand and squaring the result. Such vague definitions are contrary to the very essence of mathematics and hence should be avoided.

Especial stress was laid by Professor Miller upon the fact that dividing by 0 should be banished from elementary mathematics. The so-called indeterminate forms are really meaningless forms and it is questionable whether one should speak of evaluating such a form. As $uv = 0$ has not always for its locus the combined loci of $u = 0$ and $v = 0$, the rule relating to this case should be stated with the necessary restrictions. The fact that the last letters of the alphabet are used both for variables and for unknowns in elementary algebra has led some authors to speak of these two generally distinct concepts as if they were identical. This is the more unfortunate since the concept of a variable is continually playing a more fundamental rôle in elementary algebra. The paper has been offered to *School Science and Mathematics* for publication.

27. The problem of Professor Moulton's paper is to find the possible rôle that the near approach of the stars to one another

* Cf. Encyclopédie des Sciences mathématiques, vol. 1 (1904), p. 43.

may have played in sidereal evolution. The answer to this problem depends upon the extent of the sidereal universe, the number of stars in it, and the character of their motion. The discussion leads to the conclusion that the relatively near approaches of the stars have probably been an important factor in stellar evolution. This paper will be published by the Carnegie Institution of Washington.

The next regular meeting of the association will be held at Johns Hopkins University under the presidency of Professor T. C. Chamberlin, of Chicago University. A summer meeting will be held at Dartmouth College, beginning the last Monday in June. Professor C. J. Keyser, Columbia University, was elected vice-president and chairman of Section A, and Professor G. A. Miller, University of Illinois, was re-elected secretary. The former election is for one year and the latter for a period of five years in accord with the general rules of the association. A resolution of the council which is of especial interest to the various scientific societies is that sectional committees may dispense with a sectional program whenever an affiliated society covering the same scope meets with the section. This resolution was reaffirmed to avoid duplications of programs and to secure more hearty cooperation between the scientific societies and the association.

G. A. MILLER,
*Secretary.*

UNIVERSITY OF ILLINOIS.

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## SHORTER NOTICES.

*Memoir and Scientific Correspondence of the late Sir George Gabriel Stokes, Bart., Sc.D., etc.* Selected and arranged by JOSEPH LARMOR, D.Sc., LL.D., Sec. R.S. Two vols. 8vo. Cambridge, at the University Press.

THE history of mathematics and physics during the latter half of the nineteenth century is inseparably bound up with the career of Sir George Gabriel Stokes. As an investigator of the highest rank himself and as one who left a permanent mark on his own science, the story of his life and career is a necessary part of the development of the subject. But it is much more than this. His position as secretary of the Royal society

for over thirty years and as president for five years gave him unusual opportunities to become acquainted with those who were active in scientific work generally, and to keep in touch with all the scientific progress of his time. Of these opportunities he made the best possible use, not for himself or for his own interests, but in assisting those who wished to consult him or to obtain his advice. He was absolutely unselfish in his willingness to give his time and thought to all who wished to obtain assistance from him, and as even his first opinions on any debatable point were rarely wide of the mark, and as he would nearly always throw out some valuable suggestion or idea, the influence which he exerted cannot be easily overestimated.

Professor Larmor undertook the difficult task of gathering together and publishing the materials which should show properly these features of Stokes's character and at the same time record the inner scientific history of the time; there can be no doubt that he has achieved a notable success. The former are set forth partly in a memoir by Mrs. Laurence Humphry, the daughter of Sir George Stokes, and partly in appreciations by Professor Liveing, Sir Michael Foster, Sir William Huggins, and the Right Rev. Bishop G. F. Browne. Though the bias which necessarily accompanies the point of view of one writer is thus avoided, one cannot help wishing that the editor, as one well fitted to judge and as belonging to a later generation, had himself contributed a notice instead of confining his share of the work to selection from the materials at his disposal and to the filling in of such details as were necessary to make them intelligible or clear. The latter object, a record of the inner scientific history of the time, has been mainly achieved by the publication of numerous letters to and from various correspondents.

In the first volume the arrangement of these letters is mainly chronological, but the subjects dealt with in them are kept together as far as possible. With the careful selection of headlines the editor has made it easy for a reader to obtain the main outlines of Stokes's scientific career by merely turning over the pages. A full index enables him to find those parts which deal with any particular subject. In the second volume we are given almost entirely letters to and from correspondents arranged under the names of the latter. A striking omission — the absence of letters between Lord Kelvin and Stokes — is explained in the preface. We are glad to learn that they suffice to form a collection by themselves and that it has been

decided to publish them separately with a memorial of the lifelong friendship and collaboration of the writers.

The memoir by Mrs. Laurence Humphry is full of interesting details. In a letter to Lady Stokes before their marriage we learn how the notable paper, "On the discontinuities of the arbitrary constants which appear in divergent developments" was finally brought to a successful conclusion. Stokes writes, "I have been . . . sitting up till 3 o'clock in the morning fighting hard against a mathematical difficulty. Some years ago I attacked an integral of Airy's and after a severe trial reduced it to a calculable form. But there was one difficulty about it, which, though I tried till I almost made myself ill, I could not get over, and at last I had to give it up and profess myself unable to master it. I took it up again a few days ago, and after a two or three days fight, the last of which I sat up till 3, I at last mastered it."

In speaking of these same letters Mrs. Humphry writes, "They are remarkable also from the curious place which he assigned to his original investigations; it almost seems as if he considered them the height of dissipation and everything else a duty." One might quote almost at random from the letters to his numerous correspondents and give something worthy of notice. A considerable number of them are scientific papers of the highest value, although the results have of course been incorporated in the published memoirs of the time. But we may gather much from the tentative suggestions which may be made in a private letter but which one might not feel justified in publishing in a scientific paper. Whether we read through these volumes continuously or dip into them here and there, their perusal cannot fail to be of interest and value.

E. W. Brown.

*Computation and Mensuration.* By P. A. Lambert. The Macmillan Company, New York, 1907. ix + 92 pp.

This little volume is intended to help bridge the gap which frequently exists between the mathematics of the secondary schools offering advanced algebra and trigonometry for entrance to college and college mathematics proper. In colleges where algebra and trigonometry are taught as a part of the regular curriculum in mathematics the book might very readily be used as a review text preparatory to the work in the calculus. Opportunity to review and apply mathematical principles learned is given by

a set of 165 concrete problems of a grade which insures the arrival at definite results. The selection of the problems is based on the correct pedagogic principles that doing things promotes and sustains the students' interest, and that all work in mathematics should be consecutive.

That the text is written from the view-point of the instructor whose students are preparing for technical courses may readily be seen from the selection of the topics discussed. The following form a list representative of the contents of the ten chapters: Measurements, both direct and indirect, of lengths and areas; with special reference to the degree of accuracy possible in the computations based on the given data. Areas of rectilinear plane figures by the method of coordinates, with applications to problems in surveying. The prismoidal formula, with special reference to the computation of earthwork. Vectors, with applications to resultant forces and latitudes and departures in surveying. A chapter on the nature and applications of logarithms, which might have been strengthened by a more complete discussion of the slide rule and by the introduction of problems based on the system of natural logarithms. Approximations to the lengths of curved lines as problems in limits. Graphic algebra, including work in the solution of simultaneous equations and inequalities. Approximate areas, leading to the trapezoidal, Simpson's, and Weddle's formulas. Approximate volumes, with applications to the contents of vessels. A discussion of the results obtained by the use of the prismoidal formula when the volumes are those of revolution.

Throughout it is apparent that the author believes in the pedagogic value of accuracy, the proper arrangement of computations, and the intelligent interpretation of processes and results. It seems to the reviewer that, since the nature of the subjects considered holds the student continually responsible for concrete results, the answers to the problems should have been included in the text.

ERNEST W. PONZER.

*Anfangsgründe der darstellenden Geometrie für Gymnasien.* Von FRITZ SCHÜTTE, Oberlehrer am Gymnasium zu Düren. Leipzig, Teubner, 1905. 42 pp.

SINCE instruction in descriptive and constructive geometry was introduced into the German gymnasia in 1901, a number

of elementary texts have appeared, designed to meet the needs of the new class of pupils in this subject. The present book begins with a detailed concrete explanation of the elements of descriptive geometry. All definitions are put in bold-faced type. Solid bodies are depicted on the ground, vertical, and profile planes, and various objects are drawn to show that all three elevations are generally necessary. The straight line, plane, prism, pyramid, and regular bodies are treated in succession, a number of unsolved exercises being appended to each case. Then follow the circle, cylinder, cone, and sphere. After these ideas are mastered, the pupil is prepared to take up parallel perspective, wherein the same figures are treated again, but rather more concisely. Finally, a few pages are devoted to central perspective; methods are given for constructing a perspective picture of an object when its ground plan and profile are given. The author explains in the preface that the arrangement of subject matter is new, that heretofore too much emphasis has been laid on perspective drawing. One is tempted to feel that he has possibly gone to the other extreme, but at any rate the intelligent reader has not been bored by a multitude of details, and will finish the book thirsty for more. The typographical work is excellent; the figures are crude, but easily understood. The usefulness of this little book should not be confined to the German gymnasia.

VIRGIL SNYDER.

*Petrus Peregrinus de Maricourt and his Epistola de Magnete.* By SILVANUS P. THOMPSON, D.Sc., F.R.S. From the *Proceedings of the British Academy*, Volume II. London, published for the British Academy by Henry Frowde, Oxford University Press. 32 pp.

FOR thirty years Professor S. Thompson has made a study of the early history of magnetism, whenever opportunity presented itself for the examination of original sources. A result of this study is this article on Petrus Peregrinus and his famous Epistle on magnetism that was "done in camp at the siege of Lucera, Anno Domini 1269, the eighth day of August." No student of the history of science in the time of Peter Peregrinus and Roger Bacon can afford to overlook this article of Professor Thompson. He gives a list of the twenty-eight different ancient manuscript copies of the Epistle, tells where each is found and compares them. Two of these are in his own posses-

sion, the rest are in public libraries. "No two have yet been found to agree precisely in their text." The manuscripts in Erfurt, Vienna, Dublin, and Geneva have not yet been critically examined, hence it is still doubtful "what readings ought to be finally adopted for all the doubtful passages."

Of printed versions there are nine, or eleven if one includes a plagiarized work of 1562 and its English translation of 1579. The earliest printed version is the Augsburg tract of 1558, of which only eighteen copies are now known to exist, one of these being in the library of the American institute of electrical engineers in New York City.

FLORIAN CAJORI.

*Traite des Assurances de la Vie.* By U. BROGGI. Translated from Italian into French by S. LATTÈS. Paris, Hermann, 1907. xi + 306 pp.

THE appearance of Professor Broggi's book in a French translation makes it accessible in three languages, an edition in German having already appeared. This fact in itself is significant and bespeaks an examination of the volume.

If anyone has been accustomed to regard a text on the theory of life insurance as an unattractive compendium of formulas not all of which are carefully derived, to such a one the volume under review will by contrast be welcome. For the author has elected to present the subject as an integral part of mathematical theory, and has exhibited a proper concern regarding the development of the doctrine of probability which is the basis of the subject. He has also applied himself to the clear formulation of the various special problems which are treated and to the use of the mathematical processes in valid ways only. Even at this day successful undertakings of this sort are not so common as to be unworthy of praise.

An introductory statement of the problem of insurance is followed by the four chief divisions of the book; and of these the first contains a good discussion of the more interesting elementary topics of the theory of probability, including the theorems of Bernoulli and Poisson, an introduction to the method of least squares and a few formulas on interest. The second and third parts, constituting a considerable portion of the book, are devoted to the derivation of formulas for the principal forms of annuities and insurance upon groups of lives, the laws of mortality of De Moivre, Gompertz, and Makeham furnishing

the basis of the discussion, and for the premiums and reserves of insurance policies. Naturally these parts are devoted mainly to matters which must be treated in all complete texts on insurance. A shorter fourth part treats the theory of risk.

Good judgment has been shown by the author both in the selection of topics and in the apportionment of space to them.

Not only will the book be of particular interest to those who desire to deal with the theory of life insurance as one phase of the mathematics of statistics, but it will prove instructing to most and valuable to all of those who deal primarily with actuarial science.

G. H. LING.

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## CORRECTION.

PROFESSOR H. W. Kuhn has informed me that the theorem relating to the totality of the substitutions which are commutative with every substitution of a transitive group, published in my article in the present volume of the BULLETIN, page 19, is not new. It is found in Professor Kuhn's doctor dissertation, *American Journal of Mathematics*, volume 26 (1904), page 67.

G. A. MILLER.

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## NOTES.

THE Annual Register of the AMERICAN MATHEMATICAL SOCIETY for 1908, containing the list of officers and members of the Society, constitution and by-laws, annual reports, and catalogue of journals in the Society's library together with other accessions for 1905–1907, has recently been issued. A complete catalogue of the library up to 1905 is contained in the Register for that year, copies of which may still be obtained from the Secretary.

THE January number (volume 9, number 2) of the *Annals of Mathematics* contains the following papers: "On the classification of plane algebraic curves possessing fourfold symmetry about a point," by R. D. CARMICHAEL; "A second inverse problem in the calculus of variations," by C. E. STROMQUIST; "The continuous plane motion of a liquid bounded by two right lines," by H. C. WOLFF; "A problem in chance," by J.

K. WHITTEMORE; "The expression of constant and of alternating continued fractions in hyperbolic functions," by A. E. KENNELLY.

THE third regular meeting of the Rochester section of the association of teachers of mathematics in the middle states and Maryland was held at the University of Rochester, Saturday, February 8. The following papers were read: By W. BETZ, "The use of intuition in geometry"; by H. D. MINCHIN, "Physical demonstrations for the mathematical class-room"; by J. H. TANNER, "Genesis and extension of the number concept"; by Miss K. L. CARING, "Discussion of the syllabus of elementary algebra." The next meeting will be held at the University of Rochester on Saturday, April 11.

AT the meeting of the London mathematical society held on January 9, 1908, the following paper was read: By C. S. JACKSON, "A formula of interpolation."

THE following programme for the Fourth international congress of mathematicians, to be held at Rome in April, is announced:

Sunday, April 5, 9:30 P. M. General reception in the Aula of the University of Rome.

Monday, 10 A. M. Opening of the congress at the Capitol. 3 P. M. Election of the general bureau; award of the Guccia medal. First and second conferences.

Tuesday, Wednesday, Thursday, Friday, Saturday. 9 A. M. Sectional meetings. 3 and 3:30 P. M. Two conferences.

Organization of the sections:

I. Arithmetic, algebra, analysis: opened by ARZELÀ, CAPELLI, PASCAL, PINCHERLE.

II. Geometry: BIANCHI, SEGRE.

III. Mechanics and applied mathematics: LEVI-CIVITA, LUIGI, PIZZETTI, TOJA.

IV. Philosophic, historic, and pedagogic questions: ENRIQUES, LORIA, VAILATI.

All meetings after the opening one will be held at the Royal academy dei Lincei, Via della Lungara, 10. Reduced rates (40 to 60 per cent.) will be offered on all Italian railroads and in many hotels in Rome. All reports, conferences, and formal papers read at the Congress will be collected in the volume of

the proceedings, to be published by a committee under the management of the director of the *Rendiconti del Circolo matematico di Palermo*. Authors are requested to give their manuscripts to the general secretary before the close of the meetings.

The membership fee of 25 lire should be sent to Professor Vincenzo Reina, treasurer, Piazza S. Pietro in Vincoli 5, Rome, not later than March 25.

Subjects of conferences:

DARBOUX: "Quelques idées dans la géométrie infinitésimale."

FORSYTH: "On the present condition of partial differential equations of the second order, as regards formal integration."

HILBERT: "Die Methode der unendlich vielen unabhängigen Variabeln."

KLEIN: "Über die mathematische Encyklopädie."

LORENTZ: "Le partage de l'énergie entre la matière pondérable et l'éther."

MITTAG-LEFFLER: "Sur la représentation arithmétique des fonctions analytiques générales d'une variable complexe."

NEWCOMB: "La théorie du mouvement de la lune; son progrès et son état actuel."

PICARD: "L'analyse dans ses rapports avec la physique mathématique."

POINCARÉ: (Subject not yet announced).

VERONESE: "La geometria non archimedea."

In the inaugural session, Professor VOLTERRA will discuss "Le matematiche in Italia nella seconda metà del secolo XIX."

AT the annual public meeting of the Paris academy of sciences, held on December 2, 1907, the following prizes in mathematical sciences for 1909 and 1910 were announced; competing memoirs should be in the hands of the secretary before December 31 of the year preceding that in which the respective prizes are awarded: Francoeur prizes (fr. 1000) will be awarded annually for general progress in pure and applied mathematics; Bordin prize (fr. 3000) for 1909, see BULLETIN, volume 13, page 308; Grand prize (fr. 3000) for 1910, for the solution of the following problem: "It is known how to find all the systems of two meromorphic functions in the plane of the complex variable, connected by an algebraic relation. An analogous question arises for a system of three uniform functions of two complex variables, each having the character of a rational function in the finite part of the field, and all

connected by an algebraic function. In default of a complete solution of the problem, the academy at least desires the treatment of particular examples, leading to classes of new transcendentals."—Fourneyron prize (fr. 1000) for 1910: "Theoretic and experimental study of the blow of a battering ram in an elastic tube."—Poncelet prize (fr. 2000) for 1909 for general progress in applied mathematics; for 1910, for an important memoir in pure mathematics.—Vaillant prize (fr. 4000) for 1909: "Perfect, in some important point, the application of the dynamical principles of fluids to the theory of the propeller."—Boileau prize (fr. 1300) for 1909, for an important contribution to the theory of fluids. The remaining prizes will be awarded under the same conditions as those previously announced.

Dr. H. LEBESGUE has been promoted to a professorship of mathematical analysis at the University of Poitiers.

THE late Mr. RICHARD BROWN, of Youngstown, Ohio, left the sum of $30,000 to Mount Union College to endow the chair of mathematics. The present professor, Mr. B. F. Yanney, has been appointed to the chair.

PROFESSOR C. O. GUNTHER, of the Stevens Institute of Technology, has been promoted to be acting professor of mathematics.

PROFESSOR W. W. BEMAN, of the University of Michigan, has been granted a leave of absence during the next academic year.

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## NEW PUBLICATIONS.

### I. HIGHER MATHEMATICS.

BAKER (H. F.). Introduction to the theory of multiply periodic functions. Cambridge, 1907. 8vo. 16 + 336 pp. Cloth. 12s. 6d.

BÔCHER (M.). Introduction to higher algebra; prepared for publication with the cooperation of E. P. R. Duval. New York, Macmillan, 1908. 8vo. 11 + 321 pp. Cloth. $1.90

BROMWICH (T. J. I'A.). Quadratic forms and their classification by means of invariant factors. (Cambridge tracts in mathematics and mathematical physics.) New York, Putnam, 1907. 8vo. 8 + 100 pp. $1.10

BROZAT (W.). Ueber Scharen von $\infty^4$ Flächen im $R_3$, die durch Berührungstransformationen in Scharen von $\infty^4$ Kurven überführbar sind. Greifswald, 1907. 8vo. 53 pp. M. 1.80

BRUNO (K.). Die Grundlehren der Integral- und Differentialrechnung. Wien, Hölder, 1908. 8vo. 51 pp. M. 1.25

BUSTELLI (A. M.). Elementi di filosofia della matematica nei riguardi didascalici, con prefazione di V. Cerruti. Fasc. 1–4. Roma–Milano, Società ed. Dante Alighieri, 1905–07. 8vo. 4 fasc. Pp. 46; 54; 13 + 40; 82. L. 2.80

CERRUTI (V.). See BUSTELLI (A. M.).

CORNACCHIA (G.). Su di un metodo per la risoluzione in numeri interi dell' equazione $\sum_{k=0}^{n} C_k x^{n-k} \cdot y^k = P$. Faenza, Montanari, 1907. 4to. 15 pp.

DONADT (A.). See LÜBSEN (H. B.).

DUVAL (E. P. R.). See BÔCHER (M.).

GALILEI (G.). Unterredungen und mathematische Demonstrationen über zwei neue Wissenszweige, die Mechanik und die Fallgesetze betreffend. Aus dem Italienischen übersetzt und herausgegeben von A. von Oettingen. (Ostwalds Klassiker der exakten Wissenschaften, No. 11.) 2ter unveränderter Abdruck. Leipzig, Engelmann, 1907. 8vo. 142 pp. M. 3.00

JUNKER (F.). Repetitorium und Aufgabensammlung zur Differentialrechnung. 2te Auflage. Neudruck. Leipzig, 1907. 12mo. 129 pp. Cloth. M. 0.80

KELLER (S. S.). Analytical geometry and calculus. New York, Van Nostrand, 1907. 12mo. 359 pp. Cloth. $2.00

LARMOR (J.). See POINCARÉ (H.).

LÜBSEN (H. B.). Ausführliches Lehrbuch der analytischen oder höheren Geometrie, zum Selbstunterricht bearbeitet. 15te Auflage, neu bearbeitet von A. Donadt. Leipzig, 1907. 8vo. M. 4.00

OETTINGEN (A. VON). See GALILEI (G.).

PAGLIERO (G.). Applicazioni del calculo infinitesimale. Torino, Paravia, 1907. 8vo. 244 pp. L. 7.00

PERCIVAL (A. S.). Practical integration. For the use of engineers, etc. London, Macmillan, 1907. 8vo. 92 pp. 2s. 6d.

POINCARÉ (H.). Science and hypothesis; with a preface by J. Larmor. New York, Scribner, 1907. 12mo. 27 + 244 pp. Cloth. $1.25

SEAVER (E. P.). Mathematical handbook; containing the chief formulas of algebra, trigonometry, circular and hyperbolic functions, differential and integral calculus, and analytical geometry; together with mathematical tables. New York, McGraw, 1907. 8vo. 10 + 278 pp. Cloth. $2.50

SITTIGNANI (M.). Sulle funzioni intere di genere finito. Bologna, Zanichelli, 1907. 8vo. 8 pp.

VALENTINER (S.). Vektoranalysis. Leipzig, 1907. 12mo. 163 pp. Cloth. M. 0.80

## II. ELEMENTARY MATHEMATICS.

AMANZIO (D.). Trattato di algebra elementare. 3a edizione. Napoli, Pellerano, 1907. 8vo. 543 pp. L. 4.50

BARBISCH (H.). See JAHNE (J.).

BREMIKER (C.). Tavole logaritmico-trigonometriche con cinque decimali. Edizione italiana eseguita per cura di L. Cremona. Quinta tiratura stereotipa, con una prefazione e tavole dell' ing. A. Salmoiraghi. Milano, Hoepli, 1908. 8vo. 47 + 161 pp. L. 2.50

BRIGGS (W.) and BRYAN (G. H.). Tutorial trigonometry. 2d edition. London, Clive, 1907. 8vo. 332 pp. Cloth. 3s. 6d.

BRYAN (G. H.). See BRIGGS (W.).

BUCHANAN (A. H.). Plane and spherical trigonometry. New York, Wiley, 1907. 8vo. 6 + 96 pp. Cloth. $1.00

CREMONA (L.). See BREMIKER (C.).

ELÉMENTS de géométrie. Lignes droites; perpendiculaires et obliques; parallèles; circonférence; mesures; lignes proportionnelles; figures; polyèdres, etc. Nouvelle édition. Paris, Boulinier, 1907. 16mo. 64 pp. Fr. 0.10

FUCINI (C.). Algebra elementare per gl'istituti nautici. 3a edizione. Genova, Gioventù, 1908. 8vo. 252 pp.

——. Geometria piana e nozioni di geometria solida, per le scuole secondarie inferiori. 8a edizione. Genova, Gioventù, 1908. 8vo. 143 pp. L. 1.50

GALLI (F.). Elementi di algebra per le scuole medie di primo grado. Cremona, Fezzi, 1907. 8vo. 58 pp. L. 0.75

GHIOTTI (E.). Nozioni pratiche di aritmetica e geometria per le scuole operaie, professionali e tecniche. Nuova edizione. Torino, Gallizio, 1908. 16mo. 6 + 243 pp.

GRAVELAAR (N. L. W.). Leerboek der planimetrie. 3er druk. Groningen, 1907. 8vo. 192 pp. M. 2.50

GRÉVY (A.). Eléments d'algèbre à l'usage des élèves des classes de troisième A et première A et B (programmes de 1905). 6e édition. Paris, Vuibert, 1907. 16mo. 232 pp. Fr. 1.75

——. Géométrie théorique et pratique, conforme au programme du 27 juillet 1905, à l'usage des élèves du premier cycle B (classes de cinquième B à troisième B). 4e édition. Paris, Vuibert, 1908. 16mo. 7 + 473 pp. Fr. 3.50

——. Trigonométrie à l'usage des élèves des classes de première C et D et de mathématiques A et B (programmes de 1905). 4e édition. Paris, Vuibert, 1908. 16mo. 280 pp. Fr. 2.25

GUICHARD (C.). Traité de géométrie. Vol. I, à l'usage des classes de seconde et première C et D et mathématiques A et B. 3e édition, conforme au programme du 27 juillet, 1905. Paris, Vuibert, 1908. 8vo. 8 + 568 pp. Fr. 6.00

JACOBSTHAL (W.). See WEBER (H.).

JAHNE (J.) und BARBISCH (H.). Leitfaden der Geometrie und des geometrischen Zeichnens für Knabenbürgerschulen. Ausgegeben in 1 Bande. 2te, verbesserte Auflage. Wien, Manz, 1907. 8vo. 8 + 185 pp. M. 2.10.

KELLER (S. S.). Algebra. New York, Van Nostrand, 1907. 12mo. 113 pp. Cloth. $1.00

——. Plane and solid geometry. New York, Van Nostrand, 1907. 12mo. 212 pp. Cloth. $1.50

MAINARDI (M.). Una lezione di trigonometria. Napoli, 1907. 8vo. 8 pp.

MATRICULATION model answers. Mechanics. Sept. 1903–Sept. 1907. New edition. London, Clive, 1907. 8vo. 97 pp. 2s.

MAYER (J. E.). Mathematik für Techniker. Gemeinverständliches Lehrbuch der Mathematik für Mittelschüler sowie besonders für den Selbstunterricht. Vol. 5. Leipzig, Schäfer, 1907. 8vo. 8 + 180 pp. M. 2.40

RUDIO (F.). Des Simplicius Bericht über die Quadraturen des Antiphon und des Hippokrates, griechisch und deutsch. Mit einem historischen Erläuterungsberichte als Einleitung. Im Anhang ergänzende Urkunden, verbunden durch eine Uebersicht über die Geschichte des Problems von der Kreisquadratur vor Euklid. Leipzig, Teubner, 1907. 8vo. 10 + 184 pp. (Urkunden zur Geschichte der Mathematik im Altertume, 1es Heft.) M. 4.80

RÜEFLI (J.). Lehrbuch der ebenen Geometrie nebst einer Sammlung von Uebungsaufgaben. Zum Gebrauche an Sekundarschulen (Realschulen) und Gymnasial-Anstalten, sowie zum Selbststudium bearbeitet. 4te Auflage. Bern, Francke, 1907. 8vo. 8 + 199 pp. M. 2.40

SALMOIRAGHI (A.). See BREMIKER (C.).

SCHMIDT (A.). Beiträge zum mathematischen Unterricht: Berechnung der Logarithmen in Untersekunda; Einführung der komplexen Zahlen. Berlin, 1907. 4to. 16 pp. M. 1.00

SCHUBERT (H.). Mathematische Mussestunden. Eine Sammlung von Geduldspielen, Kunststücken und Unterhaltungsaufgaben mathematischer Natur. Kleine Ausgabe. 3te Auflage. Leipzig, Göschen, 1907. 8vo. 306 pp. Cloth. M. 5.00

VALLÍN Y BUSTILLO (A. F.). Elementos de matemáticas. Obra declarada de texto por los gobiernos de España y Portugal. Nueva edición estereotípica. Madrid, Sucesores de Hernando, 1907. 226 pp. P. 7.00

VÁSQUEZ QUEIPO (V.). Tablas de los logaritmos vulgares de los numeros 1–20000 y de las lineas trigonometricas. 24a edición. Madrid, 1907. 8vo. 268 pp. P. 5.00

VIVES Y CASADEMONT (L.). Elementos de algebra. Malaga, "La Española," 1907. 240 pp.

WEBER (H.) und WELLSTEIN (J.). Encyklopädie der Elementar-Mathematik. Ein Handbuch für Lehrer und Studierende. (In 3 Bänden.) Vol. II: Encyklopädie der elementaren Geometrie. Bearbeitet von H. Weber, J. Wellstein und W. Jacobsthal. 2te Auflage. Leipzig, Teubner, 1907. 8vo. 12 + 596 pp. Cloth. M. 12.00

WELLSTEIN (J.). See WEBER (H.).

WESTLAKE (H. F.). Construction in practical geometry. London, Philip, 1907. 8vo. 62 pp. 1s.

WHEELER (A. H.). Algebra for grammar schools, with mental exercises. Boston, Little, 1907. 12mo. 12 + 186 pp. Cloth. $0.50

——. First course in algebra; with 8000 examples, including 3000 mental exercises. Boston, Little, 1907. 12mo. 19 + 664 pp. Cloth. $1.15

## III. APPLIED MATHEMATICS.

CHOLLET (T.) et MINEUR (P.). Traité de géométrie descriptive. 1re partie à l'usage des élèves des classes de première C et D. 5e édition, conforme au programme du 27 juillet 1905. Paris, Vuibert, 1908. 8vo. 8 + 336 pp. Fr. 3.50

CONTALDI (P.). La meccanica nella scuola e nell' industria. Vol. I: Meccanica generale; meccanica applicata. 2a edizione riveduta ed ampliata. Milano, Hoepli, 1908. 8vo. 15 + 711 pp. L. 16.00

FORNER CARRATALÁ (F.). Manual de mecánica. Mecánica elemental. 2 tomos. Barcelona, 1907. 8vo. 246 and 205 pp. P. 5.00

GARNERI (A.). Corso elementare di disegno geometrico. Parte I: Problemi grafici e geometrici e ornamentazione geometrica. Fasc. 1. Torino, Paravia, 1908. 16mo. 48 pp.

GUICHARD (C.). Traité de mécanique à l'usage des élèves de mathématiques A et B et des candidats aux écoles. 5e édition. conforme au programme du 27 juillet 1905. Paris, Vuibert, 1908. 8vo. 7 + 248 pp.

HERRMANN (G.). See WEISBACH (J.).

LOVE (A. E. H.). Theoretical mechanics: an introductory treatise on the principles of dynamics, with applications and numerous examples. 2d edition. New York, Putnam, 1907. 8vo. 16 + 367 pp. Cloth. $3.00

MAURER (E. R.). Strength of materials: a practical manual of scientific methods of locating and determining stresses and calculating the required strength and dimensions of building materials. Chicago, American School of Correspondence, 1908. 8vo. 3 + 128 pp. Cloth. $1.00

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